A STATISTICAL APPROACH TO THE COMBINATION OF FORECASTS

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A STATISTICAL APPROACH TO THE COMBINATION

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To my family

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SUMMARY

This study considers situations in which several forecasts have been developed for the same series, and investigates procedures for combining these forecasts to produce a single forecast that is in some sense optimal. If $\{X_t\}$, t=1, 2, 3, . . . , represent a time series, and there are n methods that can be used to forecast this time series, such that $\hat{X}_{i,T+\tau}(T)$ is the forecast of $X_{T+\tau}$ made at origin T by forecasting method i, i=1, 2, 3, . . . , n, then the combined forecast will be represented by

$$\hat{X}_{T+\tau}(T) = \sum_{i=1}^{n} W_{i,T} \hat{X}_{i,T+\tau}(T).$$

where $\{W_i\}$ are a set of weights such that $\sum_{i=1}^{n} W_i = 1$.

The specific research objectives are to find an appropriate procedure for estimating and updating the weights in a system that considers several forecasts to produce a single forecast of a future observation; to test the effectiveness of this procedure by using industrial and/or economic time series with a computer simulation experiment and to develop guidelines for deciding when it is desirable to combine several different forecasts and when it is desirable to use the best individual forecast.

It is shown that under certain conditions, a linear combination of individual forecasts will produce one forecast that is optimal in a minimum mean square error sense. Methods for estimating and updating the weights in this linear combination are developed. Examples and comparisons of the procedures with others suggested in the literature are given.

CHAPTER I

INTRODUCTION

1.1 Background

Forecasting plays a major role in many fields, including production and operations management, economics, and governmental operations. For example, in production and operations management, forecasts of future demand for the organization's products are required to drive the other components of the management information system, such as procurement, production planning, inventory control, and distribution planning.

Forecasting methods can be generally classified as qualitative or quantitative. Qualitative methods usually involve subjective analysis by experts in the field to arrive at a forecast. An example of such a procedure is the Delphi Method. Other qualitative approaches include estimates of sales and the use of management experience. Sometimes historical data may be considered, but the process used to generate the forecast is subjective.

Quantitative forecasting methods make formal use of mathematical and statistical procedures to produce the forecast. The general procedure is to examine an appropriate set of historical data, determine the correct underlying model for the process, estimate the parameters of this model by statistical techniques, and then extrapolate the model into the future. The assumption made in the use of quantitative forecasting methods is that a stable pattern will remain stable over the forecast lead time.

There are many different types of quantitative forecasting techniques. Broadly speaking, these methods may be classified into three general categories: smoothing, time series methods, and causal methods.

Smoothing methods use historical data to obtain a set of smoothed statistics for the series, which are used to generate the forecast for the required future period. Under this category we find moving average methods [1], [2], [3], [4], single exponential smoothing [1], [3], [5], [6], exponential smoothing for linear trends (Brown's Method [3], [5], [6] and Holt's Two Parameter Smoothing [3], [7], [8]), general exponential smoothing [3], [6], Winters' Method [1], [3], [7], [8] which incorporates a seasonal as well as a trend adjustment, and adaptive filtering which determines and updates the optimal weights to be applied to past data to minimize the squared errors [3].

Time series models usually explicitly assume that some autocorrelative structure exists in the series, and then take account of this autocorrelation in modeling the time series and forecasting. Box and Jenkins' book [9] is an important contribution to time series analysis, forecasting and control. Other references for further study include [1], [3], [9], [10] and [11].

Causal models relate the value of a variable of interest to several other variables, assuming that the variable of interest exhibits a cause-effect relationship with the other variables. These models try to discover the relationship so that forecasts can be found using the values of the causal variables. The major methods under these models are multiple regression techniques, econometric models, and multivariate time series models [1], [3], [11], [12], [13].

Frequently an analyst may find that there are several different approaches that could be used to model and forecast a particular time series. However, the choice between these methods may not be obvious. For example, one may find that both Winters' method and a seasonal ARIMA process are adequate models of the time series, but one may not dominate the other. Instead of discarding one model and using the other, one possible approach is to combine both forecasts in some fashion to produce a single forecast that is in some sense superior to either individual forecast. This research is concerned with the general problem of combining two or more forecasts of the same time series into a single forecast.

1.2 Research Objectives

The general research objective is to investigate and develop a methodology for combining several forecasts derived from quantitative techniques to produce a single forecast that has better statistical properties than the individual forecasts. A well known result in estimation theory can be applied to this problem. Suppose that there are two independent random variables $(\hat{\theta}_1, \hat{\theta}_2)$ that are unbiased estimators of a common parameter, say θ, and if

$$V(\hat{\theta}_1) = \sigma_1^2$$

$$V(\hat{\theta}_2) = \sigma_2^2$$

where V(.) is the variance of the estimator, then

$$\hat{\theta} = \alpha\, \hat{\theta}_1 + (1 - \alpha)\, \hat{\theta}_2$$

is an unbiased estimator of θ that has

$$V(\hat{\theta}) < V(\hat{\theta}_1) \quad \text{and}$$

$$V(\hat{\theta}) < V(\hat{\theta}_2)$$

where

$$\alpha = \frac{\sigma_2^{\,2}}{(\sigma_1^{\,2} + \sigma_2^{\,2})}$$

is the weight factor.

Various authors have investigated the use of weight functions based on the above result in the combination of forecasts. Their methods for choosing the weights are heuristic. In this research, a new method for estimating and updating the weights in the linear combination of forecasts will be investigated. The specific research objectives are:

1. To find an appropriate procedure for estimating and updating the weights in a system that combines several forecasts to produce a single forecast of a future observation.
2. To test the effectiveness of this procedure by using industrial and/or economic time series, with a computer simulation

experiment. This test will compare the procedure with the use of individual forecasting methods, and with other forecast combination procedures suggested in the literature.

3. To develop guidelines for deciding when it is desirable to combine several different forecasts, and when it is desirable to use the best individual forecast.

1.3 Method of Approach

A review of the existing literature in forecasting with emphasis on the combination of forecasts will be presented in Chapter II. Chapter III deals with certain statistical properties of the combination of forecasts. A theoretical basis for the determination of weights and weight updating are presented in this chapter.

To test the effectiveness of the procedure to estimate and update the weights for combining several forecasts, real time series were used. The simulation experiment as well as the results are presented in Chapter IV.

Chapter V presents an analysis of the results obtained from the simulation experiment.

Finally, Chapter VI presents a summary of the research objectives and results, conclusions, and suggested areas for further research.

CHAPTER II

LITERATURE SURVEY

2.1 Introduction

This chapter will review the existing literature on the combination of forecasts. It will also give a brief overview of the general types of forecasting methods used in this research.

2.2 The Combination of Forecasts

2.2.1 Notation. Let $\{x_t\}$, t=1, 2, 3. . . , represent a time series. Suppose that n methods can be used to forecast this time series, and let $\hat{X}_{i,T+\tau}(T)$ be the forecast of $X_{T+\tau}$ made at origin T by forecasting and method i, i=1, 2, . . . ,n. We would like to find some combination of of the n forecasts produced by the individual methods such that the combined forecast is "best" in the sense of having a small forecast error variance. That is,

$$\hat{X}_{T+\tau}(T) = \sum_{i=1}^{n} W_{i,T} \hat{X}_{i,T+\tau}(T) \tag{1}$$

is the combined forecast, and the forecast error is

$$e_{T+\tau}(T) = X_{T+\tau} - \hat{X}_{T+\tau}$$

$$= \hat{X}_{T+\tau} - \sum_{i=1}^{n} W_{i,T} \hat{X}_{i,T+\tau}(T)$$

Properties of the combined forecast depend on the weights $\{w_{i,T}\}$.

2.2.2 Related Literature. Various authors have studied the combination of forecasts. Generally, they have used heuristic methods to determine the appropriate weights.

Bates and Granger [14] studied the combination of two separate sets of forecasts using past forecast errors in the original series to determine the weights. Their primary conclusion was that the composite set of forecasts can yield lower mean-square error than either of the original forecasts. Their work emphasized the different ways in which forecasts can be combined and proposed several methods for determining the weights. The first method suggested by Bates and Granger for combining two forecasts was to weight each forecast equally. That is, $W_1 = W_2 = 0.5$ They showed by a numerical example that the variance of forecast error for the combined forecast with equal weights for each of the individual forecasts is smaller than the individual forecast error variances. Bates and Granger then presented a method for choosing the weights designed to produce a minimum forecast error variance for the combined forecast. Their work is based on the following assumptions:

1. The performance of the individual forecasts is consistent with time such that the variance is not changing with t.
2. Both forecasting methods yield unbiased forecasts.
3. The weights add to 1, thereby producing an unbiased forecast. That is, the combined forecast $\hat{X}_{T+\tau}(T)$ is unbiased if $E[\hat{X}_{T+\tau}(T)] = E[X_{T+\tau}(T)]$. From Equation (1)

$$\hat{X}_{T+\tau}(T) = \sum_{i=1}^{n} W_{i,T} \hat{X}_{i,T+\tau}(T).$$

Then,

$$E[X_{T+\tau}(T)] = E\left[\sum_{i=1}^{n} W_{i,T} \hat{X}_{i,T+\tau}(T)\right]$$

$$= \sum_{i=1}^{n} W_{i,T} E[\hat{X}_{i,T+\tau}(T)]$$

but

$$E[\hat{X}_{T+\tau}(T)] = E[X_{T+\tau}] \text{ for all } i$$

then

$$E[\hat{X}_{T+\tau}(T)] = E[X_{T+\tau}] \sum_{i=1}^{n} W_{i,T}.$$

Then for $\hat{X}_{T+\tau}(T)$, the combined forecast, to be unbiased, we must have

$$\sum_{i=1}^{n} W_{i,T} = 1. \tag{2}$$

For two forecasting methods, Bates and Granger [14] showed that the variance of the combined forecast error, $V[e_{T+\tau}(T)]$, can be written as

$$V[e_{T+\tau}(T)] = W_{1,T}^{2}\ \sigma_1^{2} + (1-W_{1,T})^{2}\ \sigma_2^{2} + 2\rho W_{1,T}\ \sigma_1\ (1-W_{1,T})\ \sigma_2 \qquad (3)$$

where $W_{1,T}$ is the weight given to the first method at period T, ρ is the correlation coefficient between forecasts errors in the first set of forecasts and those in the second set, and σ_1^2 and σ_2^2 are the variances of forecasts errors of the two individual forecasts. To minimize the combined forecast error variance, they obtain as the optimal value for $W_{1,T}$ the following expression

$$W_{1,T} = \frac{\sigma_2^{2} - \rho\ \sigma_1\ \sigma_2}{\sigma_1^{2} + \sigma_2^{2} - 2\rho\sigma_1\ \sigma_2}\ . \qquad (4)$$

Since in practice one never knows the real values of σ_1^2, σ_2^2, and ρ, Bates and Granger developed five different heuristic methods to compute the weights. Let $\{e_{i,T}\}$, i=1, 2 be the two series of forecast errors from the different forecasting methods. The five methods of computing the weights $W_{1,T}$ and $W_{2,T}$ are as follows:

$$1.\quad W_{1,T} = \frac{\sum_{j=T-\nu}^{T-1} (e_{2,j})}{\sum_{j=T-\nu}^{T-1} (e_{i,j})^2 + \sum_{j=T-\nu}^{T-1} (e_{2,j})^2} \qquad (5)$$

where ν is a constant such that $\nu \geq 1$.

2. $$W_{1,T} = \alpha\, W_{1,T-1} + (1-\alpha)\ \frac{\sum_{j=T-\nu}^{T-1} (e_{2,j})^2}{\sum_{j=T-\nu}^{T-1} (e_{i,j})^2 + \sum_{j=T-\nu}^{T-1} (e_{2,j})^2} \qquad (6)$$

where α is a constant such that $0 \leq \alpha \leq 1$.

3. If $S_i^2 = \sum_{t=1}^{T-1} k^t (e_{i,t})$ where k is an arbitrarily chosen constant or weight which for k>1 gives more weight to recent error variances than to distant ones:

$$W_{1,T} = \frac{S_2^2}{S_1{}^2 + S_2{}^2}$$

4. If C is a weighted covariance, where

$$C = \sum_{t=1}^{T-1} k^t\ e_{1,t}\ e_{2,t},$$

then

$$W_{1,T} = \frac{S_2^2 - C}{S_1^2 + S_2^2 - 2C} \qquad (8)$$

5. $$W_{1,T} = \alpha\, W_{1,T-1} + (1-\alpha)\ \frac{e_{2,T-1}}{e_{1,T-1} + e_{2,T-1}} \qquad (9)$$

Granger and Newbold [15] extended and modified the suggested weights in reference [14] to the general case of n forecasting methods. Formulas (5) to (9) were generalized as follows for n equal to the number of forecasting methods to be combined

Formula 1
$$W_{i,T} = \frac{\left(\sum_{t=T-\nu}^{t-1} e_{i,t}^{2}\right)^{-1}}{\sum_{j=1}^{n}\left(\sum_{t=T-\nu}^{T-1} e_{j,t}^{2}\right)^{-1}}, \quad \nu \geq 1 \tag{10}$$

Formula 2
$$\underline{W}_T = \left(\hat{\Sigma}^{-1}\,\underline{1} \;/\; \underline{1}'\,\Sigma^{-1}\,\underline{1}\right)$$

subject to $0 \leq W_{i,T} \leq 1$ for all i,

$$\left(\hat{\Sigma}\right)_{ij} = \frac{1}{\beta}\sum_{t=T-\beta}^{T-1} e_{i,t}\; e_{j,t}. \tag{11}$$

Formula 3
$$W_{i,T} = \gamma\, W_{i,T-1} + (1-\gamma)\; \frac{\left(\sum_{t=T-\nu}^{T-1} e_{i,t}^{2}\right)^{-1}}{\sum_{j=1}^{n}\left(\sum_{t=T-\nu}^{T-1} e_{j,t}^{2}\right)^{-1}}, \quad 0<\gamma<1 \tag{12}$$

Formula 4
$$W_{i,T} = \frac{\left(\sum_{t=1}^{T-1} \theta^{t}\, e_{i,t}^{2}\right)^{-1}}{\sum_{j=1}^{n}\left(\sum_{t=1}^{T-1} \theta^{t}\, e_{j,t}^{2}\right)^{-1}}, \quad \theta \geq 1 \tag{13}$$

Formula 5 $$\underline{W} = \left(\hat{\Sigma}^{-1}\,\underline{1}\right) / \left(\underline{1}'\,\hat{\Sigma}^{-1}\,\underline{1}\right)$$

subject to $0 \leq W_{i,T} \leq 1$ for all i,

$$\left(\hat{\Sigma}\right)_{ij} = \frac{\sum_{t=1}^{T-1} \phi^t\, e_{i,t}\, e_{j,t}}{\sum_{t=1}^{T-1} \phi^t}, \quad \phi \geq 1 \tag{14}$$

Granger and Newbold [15] investigated the performance of the five weight determination procedures. Three interesting conclusions can be derived from this paper

1. Combining forecasts is generally worthwhile since it often leads to a reduction in forecast error variance.
2. Those combining methods that ignore the correlative structure of the forecast errors (Method 1,3 and 4) are more successful than those that consider correlation (Method 2 and 5).
3. The combined forecast

$$\hat{X}_{T+\tau}(T) = W_{1,T}\,\hat{X}_{1,T+\tau}(T) + (1 - W_{1,T})\,\hat{X}_{2,T+\tau}(T)$$

performs well in a wide range of situations if the weights are defined as

$$W_{1,T} = \frac{\sum_{t=T-12}^{T-1} e_{2,t}^2}{\sum_{t=T-12}^{T-1} \left(e_{1,t}^2 + e_{2,t}^2\right)}.$$

Dickinson [16] considered the sampling distributions of the weights used for the individual forecasts and the error variance of the combined forecast. He extended the results of Bates and Granger [14] to the combination of n forecasts which exhibit no covariance between their errors. Dickinson proved that the error variance of the combined forecast follows a Wishart Distribution* $W_n(m,\Sigma)$, where n is equal to the number of forecasting methods in the combination, m is the number of forecasts in each forecasting method and Σ is the covariance matrix. Σ_{ij} is the covariance between the error of forecast i and forecast j at a particular point in time. Dickinson proved that in the absence of correlation, the weights used in the combined forecast follow a Beta Distribution. His study concludes that it is doubtful whether combined forecasts offer much improvement over individual forecasts because of the unreliability of the weight estimates.

Bunn [17] presented a Bayesian approach to the combination of forecasts. He uses subjective probabilities for the determination of the posterior distribution for the number of times, k, one method outperforms the others where one forecast can be considered to outperform another if it has a smaller absolute error. He found that k follows a Beta Distribution. The optimal weights for the combination of two forecasts result to be the posterior mean of k, $\bar{k}$, and $(1-\bar{k})$ respectively. He extended the results to combination of n forecasts, using the Dirichlet Distribution. Bunn does not test the performance of his approach or compare it with other methods.

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*The Wishart Distribution is a multivariate generalization of the χ^2 distribution. See [21].

None of the methods described has solved the problem of the determination of optimal weights for the combination of forecasts.

2.3 Forecasting Methods

The following section gives a general description of the five forecasting methods used in this research: adaptive filtering, Box-Jenkins (ARIMA) models, exponential smoothing, Winters' Method, and linear regression.

Each method is based on certain general concepts and assumptions. These assumptions often limit the method's application. The limiting factors or characteristics of a forecasting method can be summarized as follows:

1. Pattern of data: Broadly speaking, a series can be either stationary or nonstationary. It can have other identifiable characteristics such as trend or seasonality. Since the purpose of a time series forecasting method is to identify and model the pattern, it is important to know what forecasting methods are appropriate for the specific pattern.
2. Time horizon or lead time: Little is known about the performance of different forecasting methods for different lead times. If it were known that one forecasting method is dominant over a specific range of lead times, then this would often simplify the modeling problem. It is expected, in general, that forecast accuracy decreases as lead time increases.
3. Accuracy: One of the basic selection criteria for the forecasting method is its accuracy in both fitting and

forecasting the data. Some forecasting methods may be more accurate than others, but this increased accuracy is usually accomplished by increased costs of model development and system operation.

4. Amount of data: Another factor to consider is the amount of historical data available. Some methods require a large amount of data for model development, while others do not.

2.3.1 Exponential Smoothing Methods. Exponential smoothing techniques are among the most popular and widely used forecasting methods because of their simplicity, and their relatively modest data requirements. These technique can be applied to short or immediate term predictions of a large number of items. Single and multiple exponential smoothing methods are used in this research.

Single exponential smoothing [1], [3], [5], [6] is based on smoothing historical data of a time series in an exponential manner. If we let S_t be the smoothed statistic for period t for the variable X made at period t, and α a constant that assumes any value between 0 and 1, known as the smoothing constant, then using single exponential smoothing the forecast S_{t+1} is given by

$$S_t = \alpha X_t + (1 - \alpha) S_{t-1}$$

By successively substituting it is possible to show that

$$S_t = \alpha \sum_{t=1}^{\infty} (1 - \alpha)^{t-1} X_t$$

That is, historical values of the variable of interest X_t are weighted in an exponential manner. The forecast function becomes

$$X_{t+\tau}(t) = S_t, \; t = 1, 2, \ldots$$

This technique often works well when applied to series having a constant mean or when mean changes very little during time.

Exponential smoothing can also be used in cases where the series exhibits a significant linear trend. In these models, the computational technique is similar to the single exponential smoothing, but one additional factor is included to estimate the trend. In general, we have the time series model

$$X_t = b_1 + b_2 t + e_t$$

The double exponential smoothing equations are as follows:

$$S_t = \alpha X_t + (1 - \alpha) S_{t-1}$$

$$S_t^{[2]} = \alpha X_t + (1 - \alpha) S_{t-1}^{[2]}$$

Forecasts of future observations, say at period T, would be obtained from

$$\hat{X}_{T+\tau}(t) = \left[2 + \frac{\alpha\tau}{(1-\alpha)} \right] S_t - \left[1 + \frac{\alpha\tau}{(1-\alpha)} \right] S_t^{[2]}$$

In general, exponential smoothing of order k can be used to estimate the coefficients in a polynomial of degree k-1. The forecast function can be expressed as a linear combination of the k exponentially smoothed statistics. For computational detail, see [6].

An important advantage of the exponential smoothing techniques is that little historical data is required to generate a forecast. Furthermore, only a limited amount of historical data must be carried by the information processing system. The value of the smoothing constant, the most recent values of the smoothed statistics, and the most recent observation are required only.

2.3.2 Winters' Method of Forecasting. Winters' Method [1], [3], [7] is an extension of the linear exponential techniques to account for the seasonality in the data. The underlying model is assumed to be

$$X_t = (b_1 + b_2 t)C_t + e_t$$

where

b_1 = the permanent component, estimated using

$$\hat{a}_1(T) = \frac{\alpha X_T}{\hat{C}_T(T-1)} + (1 - \alpha) \left[\hat{a}_1(T-1) + \hat{b}_2(T-1)\right]$$

$0 < \alpha < 1$ is a smoothing constant,

L is the length of the season,

b_2 = a linear trend component estimated by

$$\hat{b}_2(T) = \beta \left[\hat{a}_1(T) - \hat{a}_1(T-1) \right] + (1 - \beta)\ \hat{b}_2(T-1)$$

where $0 < \beta < 1$ is a second smoothing constant,

C_t = multiplicative seasonal factor, estimated using

$$\hat{C}_T(T) = \frac{\gamma X_T}{a_1(T)} + (1 - \gamma)\ \hat{C}_T\ (T-L)$$

where $0 < \gamma < 1$ is a third smoothing constant,

e_t = random error component.

The main advantage of this method is that it incorporates a seasonal as well as a trend adjustment to the model. It is expected to perform well on those series exhibiting both trend and seasonal variation.

2.3.3 Linear Regression. Linear Regression analysis can be used for a process in which the mean changes linearly with time. Here the independent variable is time, and the dependent variable is the time series under study. The method of least squares is used to estimate the parameters in the underlying time series model. A number of properties of this technique are presented in Montgomery and Johnson [1].

2.3.4 Fundamentals of Adaptive Filtering. The method of adaptive filtering was developed in the field of systems engineering, [18]. It uses a weighted sum of past observations as a forecast. That is,

$$\hat{X}_{t+1} = W_1 X_t + W_2 X_{t-1} + W_3 X_{t-2} + \cdot \cdot \cdot + W_n X_{t-n+1}$$

where

X_{t+1} = forecast for the period t+1

W_i = weight to be assigned to observation t-i+1

X_i = observed value in period t

n = total number of weights.

The method of estimating the weights is based on the forecast errors

$$e_{t+1} = X_{t+1} - \hat{X}_{t+1}$$

The updated vector of weights will be W_i'

$$W_i' = W_i + 2ke_{t+1} X_{t-i+1}$$

where k is a learning constant. For this method, the user has to specify the value of k as well as the number of weights to be used.

Adaptive filtering is a relatively new method. It has been used in this thesis for the purpose of studying its behavior, and comparing it to other methods.

2.3.5 Some Basic Concepts of Box-Jenkins (ARIMA) Methods. Box and Jenkins or ARIMA models are commonly used when complex time series are under study, and we wish to take explicit account of the autocorrelative structure of the data. This method assumes no specific pattern or model for the data. The model to describe the data is developed from an analysis of autocorrelation and partial autocorrelation functions. Time Series are classified as stationary and nonstationary. Stationary series behave as if they have a constant mean, while nonstationary series tend to drift, behaving as if the mean (or perhaps other moments) changes with time. An example of a stationary and nonstationary time series is shown in Figure 2.1.

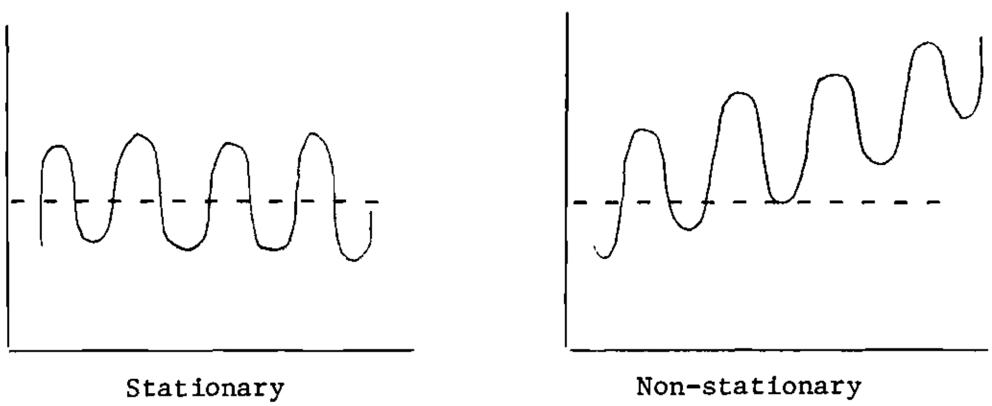


Figure 2.1 Example of stationarity and nonstationarity

Any stationary time series can be described with a model of the form

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots \phi_p X_{t-p} + e_t$$

$$- \theta_1 e_{t-1} - \theta_2 e_{t-2} - \ldots \theta_q e_{t-q}$$

This is known as an autoregressive moving-average model (ARMA) of order (p,q).

When the series is non-stationary, successive differences are used to convert the series to stationarity. The operator ∇ is defined as

$$\nabla X_t = X_t - X_{t-1}$$

A model representing the class of non-stationary time series in known as the autoregressive integrated moving average process (ARIMA) of order (p, d, q), or

$$\Phi_p (B) \nabla^d X_t = \Theta_q (B) e_t$$

where Φ_p = autoregressive operator of order p,
$\Phi_p(B) = 1-\phi_1 B-\phi_2 B^2 -\ldots - \phi_p B^p$

Θ_q = moving average operator of order q,
$\Theta_q(B) = 1-\Theta_1 B^1-\Theta_2 B^2 -\ldots-\Theta_q B^q$

B = Backward shift operator, defined such that $BX_t = X_{t-1}$

The procedure for using Box-Jenkins models starts with the identification of a tentative model. This is accomplished by the examination of the autocorrelation and partial autocorrelation coefficients. Then the parameters in the tentative models are estimated using standard statistical estimation methods. A test of the models adequacy called diagnostic checking, is performed next. For this step a series of residuals

$$e_t = X_t - \hat{X}_t$$

are studied using autocorrelation and partial autocorrelation functions. The existence of a pattern is an indication of a bad model, but if errors are random-(white noise) the model is adequate and we can proceed to the forecasting phase.

Box-Jenkins models can be shown to provide forecasts that are optimal in a minimum mean square error sense. Their main disadvantage is that they require elaborate modeling techniques, and their application may not be economically justified.

CHAPTER III

A STATISTICAL APPROACH TO THE COMBINATION OF FORECASTS

3.1 Introduction

This chapter will study the statistical properties of the combination of forecasts. A new method for weight estimation and updating will be presented.

3.2 Notation

Chapter II, section 2.2.1 includes a discussion of the notation to be used for the combination of forecasts. The combined forecast was presented as

$$\hat{X}_{T+\tau}(T) = \sum_{i=1}^{n} W_i \, X_{i,T+\tau}$$

where the $\{W_i\}$ are the set of weights, and $X_{i,T+\tau}$ the forecast of $X_{T+\tau}$ made at origin T by forecasting method i, i=1, 2, . . . , n.

Several forecasting methods can be used to model a particular time series. Each method can be compared to the other by its accuracy in both fitting the data and forecasting the future. Several criteria of model's accuracy are available. Mean Square Error (MSE) will be used as the main basis for comparison in this research.

$$\text{MSE} = \frac{\sum_{t=1}^{T} (X_i - \hat{X}_i)^2}{T-1}$$

Other measurements of model's accuracy may include

$$\text{Average forecase error:} \quad \bar{e} = \frac{\sum_{i=1}^{T} e_i}{T} \quad ,$$

$$\text{Variance of forecast error:} \quad \hat{V}(e) = \frac{\sum_{i=1}^{T} e_i^{\,2} - \frac{\sum_{i=1}^{T} e_i}{T}}{T-1} \quad \text{, and}$$

$$\text{Standard deviation of forecast error:} \quad \hat{\sigma}_e = \sqrt{\hat{V}(e)} \; .$$

3.3 Properties of the Combined Forecast

It has been stated before that the properties of the combined forecast depend on the weights $\{w_t\}$. If the individual forecasts $\{\hat{X}_{i,T+\tau}(T)\}$ are unbiased, then for $\hat{X}_{T+\tau}(T)$ the combined forecast, to be unbiased, we must have

$$\sum_{i=1}^{n} w_{i,T} = 1.$$

A reasonable approach in choosing the weights will be to select them proportional to the variances of the individual forecast errors. Let $\sum_{T+\tau}$ be the covariance matrix of the forecast errors at period T+τ

$$\sum_{T+\tau} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 & . & . & . & \sigma_{1n}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 & . & . & . & \sigma_{2n}^2 \\ . & . & . & & & & . \\ . & . & . & & & & . \\ . & . & . & & & & . \\ \sigma_{n1}^2 & \sigma_{n2}^2 & \sigma_{n3}^2 & & & & \sigma_{nn}^2 \end{bmatrix}$$

where σ_{ij}^2 is the covariance of forecast errors for method i and method j and σ_{ii}^2 is the variance of forecast errors for method i, period T+τ.

Assuming a combination of only two methods, let $X_{i,T+\tau}(T)$ be the forecast of $X_{T+\tau}$ made at origin T by forecasting method i, i=1, 2. Let

$e_{i,T+}(T)$ be the error for forecasting method i, i=1, 2, and

$$E(e_{i,T+\tau}(T)) = 0,\ E(e_{i,T+\tau}(T)^2) = \sigma_i^2 = v(e_{i,T+\tau}(T))$$

The covariance of forecast errors for the two methods is

$$E(e_{1,T+\tau}(T)\ e_{2,T+\tau}(T)) = \sigma_{12}^2 = \rho\sigma_1\ \sigma_2$$

where ρ = correlation coefficient

σ_i = standard deviation of forecast errors for forecasting method i.

The combined forecast will be

$$\hat{X}_{T+\tau}(T) = W_{1,T}\ \hat{X}_{1,T+\tau}(T) + W_{2,T}\ \hat{X}_{2,T+\tau}(T)$$

where, to be unbiased, we required $W_{1,T} + W_{2,T} = 1$. The forecast error for the combination is

$$\begin{aligned}
e_{T+\tau}(T) &= X_{T+\tau}(T) - \hat{X}_{T+\tau}(T) \\
&= X_{T+\tau} - W_{1,T}\ \hat{X}_{1,T+\tau}(T) - W_{2,T}\ \hat{X}_{2,T+\tau}(T) \\
&= W_{1,T}(X_{T+\tau} - \hat{X}_{1,T+\tau}(T)) + W_{2,T}(X_{T+\tau} - \hat{X}_{2,T+\tau}(T)) \\
&= W_{1,T}\ e_{1,T+\tau}(T) + W_{2,T}\ e_{2,T+\tau}(T)
\end{aligned}$$

For two methods the error variance becomes

$$V[e_{T+\tau}(T)] = V[W_{1,T}\ e_{1,T+\tau}(T) + W_{2,T}\ e_{2,T+\tau}(T)]$$

$$= V[W_{1,T}\ e_{1,T+\tau}(T)] + V[W_{2,T}\ e_{2,T+\tau}(T)]$$

$$+ 2\ \mathrm{Cov}\ [W_{1,T}\ e_{1,T+\tau}(T),\ W_{2,T}\ e_{2,T+\tau}(T)] \qquad (15)$$

where $\mathrm{Cov}\ [W_{1,T}\ e_{1,T+\tau}(T),\ W_{2,T}\ e_{2,T+\tau}(T)] =$

$$= E\left\{[W_{1,T}\ e_{1,T+\tau}(T) - E(W_{1,T}\ e_{1,T+\tau}(T))][W_{2,T}\ e_{2,T+\tau}(T) - E(W_{2,T}\ e_{2,T+\tau}(T))]\right\}$$

$$= E\left\{[W_{1,T}\ e_{1,T+\tau}(T) - W_{1,T}(0)][W_{2,T}\ e_{2,T+\tau}(T)]\right\}$$

$$= E\left\{[W_{1,T}\ W_{2,T}\ e_{1,T+\tau}(T)\ e_{2,T+\tau}(T)\right\}$$

$$= W_{1,T}\ W_{2,T}\ \ E(e_{1,T+\tau}(T)\ e_{2,T+\tau}(T))$$

$$= W_{1,T}\ W_{2,T}\ \sigma_{12}^2$$

$$= W_{1,T}\ W_{2,T}\ \rho\ \sigma_1\ \sigma_2 \qquad (16)$$

By substitution in equation (15)

$$V[e_{T+\tau}(T)] = W_{1,T}^2\ \sigma_1^2 + W_{2,T}^2\ \ \sigma_2^2 + 2\ W_{1,T}\ W_{2,T}\ \rho\ \sigma_1\ \sigma_2$$

This result was derived by Bates and Granger [14]. Since our objective is to minimize variance, if we take the partial derivative of $V(e_{T+\tau}(T))$ with respect to weight $W_{1,T}$, we have

$$\frac{\delta V[e_{T+\tau}(T)]}{\delta W_{1,T}} = 2\, W_{1,T}\, \sigma_1{}^2 - 2\,(1-W_{1,T})\, \sigma_2^2 + 2\rho\sigma_1\, \sigma_2 - 4\, W_{1,T}\, \rho\sigma_1\, \sigma_2 \;=\; 0$$

$$2\, \sigma_2{}^2 - 2\,\rho\,\sigma_1\, \sigma_2 = W_{1,T}\, (2\, \sigma_1{}^2 + 2\, \sigma_2{}^2 + 4\,\rho\,\sigma_1\, \sigma_2) \tag{17}$$

As a result:

$$W_{1,T}(T) = \frac{\sigma_2{}^2 - \rho\,\sigma_1\, \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\,\rho\,\sigma_1\, \sigma_2} \tag{18}$$

This result was first presented by Bates and Granger [14]. By substitution in equation (17), we find that

$$V[e_{T+\tau}(T)] = W_{1,T}^{\;2}\, \sigma_1{}^2 + W_{2,T}^{\;2}\, \sigma_2{}^2 + 2\, W_1\, W_2\, \rho\, \sigma_1\, \sigma_2$$

$$= \left(\frac{\sigma_2{}^2 - \rho\,\sigma_1\sigma_2}{\sigma_1{}^2 + \sigma_2^2 - 2\,\rho\,\sigma_1\sigma_2}\right)^2 \sigma_1{}^2 + \left[1 - \left(\frac{\sigma_2^2 - \rho\,\sigma_1\sigma_2}{\sigma_1{}^2 + \sigma_2^2 - 2\,\rho\,\sigma_1\sigma_2}\right)\right]^2 \sigma_2^2$$

$$+ 2\left(\frac{\sigma_2{}^2 - \rho\,\sigma_1\sigma_2}{\sigma_1{}^2 + \sigma_2^2 - 2\,\rho\,\sigma_1\sigma_2}\right)\left[1 - \left(\frac{\sigma_2{}^2 - \rho\,\sigma_1\sigma_2}{\sigma_1{}^2 + \sigma_2^2 - 2\,\rho\,\sigma_1\sigma_2}\right)\right] \rho\, \sigma_1\sigma_2$$

$$= \frac{(\sigma_2{}^2-\rho\sigma_1\sigma_2)^2\ \sigma_1{}^2+(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2 - \sigma_2^2 + \rho\sigma_1\sigma_2)^2\sigma_2{}^2}{(\sigma_1{}^2+\sigma_2^2 - 2\rho\sigma_1\sigma_2)^2}$$

$$+ \frac{2(\sigma_2{}^2-\rho\sigma_1\sigma_2)(\sigma_1{}^2+\sigma_2{}^2-2\ \rho\sigma_1\sigma_2 - \sigma_2{}^2+ \rho\sigma_1\sigma_2)\ \rho\ \sigma_1\sigma_2}{(\sigma_1{}^2+\sigma_2^2 - 2\rho\sigma_1\sigma_2)^2}$$

$$= \frac{(\sigma_2{}^2-\rho\sigma_1\sigma_2)^2\sigma_1{}^2+(\sigma_1{}^2-\rho\sigma_1\sigma_2)^2\sigma_2^2+ 2(\sigma_2{}^2-\rho\sigma_1\sigma_2)(\sigma_1{}^2-\rho\sigma_1\sigma_2)\rho\sigma_1\sigma_2}{(\sigma_1{}^2+\sigma_2^2 - 2\rho\sigma_1\sigma_2)^2}$$

$$= [(\sigma_2^4-2\rho\sigma_1\sigma_2{}^3+\rho^2\sigma_1{}^2\sigma_2{}^2)\sigma_1{}^2+(\sigma_1{}^4-2\rho\sigma_1{}^3\sigma_2+\rho^2\sigma_1{}^2\sigma_2{}^2)\ \sigma_2{}^2$$

$$+\ 2\rho\sigma_1{}^3-2\rho^2\sigma_1{}^2\sigma_2{}^4-2\rho^2\sigma_1{}^4\sigma_2{}^2+2\rho^3\sigma_1{}^3\sigma_2{}^3]/(\sigma_1{}^2+\sigma_2^2 - 2\rho\sigma_1\sigma_2)^2$$

$$= (\sigma_2{}^4\sigma_1{}^2-2\rho\sigma_1{}^3\sigma_2{}^3+\rho^2\sigma_1{}^4\sigma_2{}^2 + \sigma_1{}^4\sigma_2{}^2- 2\rho\sigma_1{}^3\sigma_2{}^3+\rho^2\sigma_1{}^2\sigma_2{}^4 +$$

$$+\ 2\rho\sigma_1{}^3\sigma_2{}^3-2\rho^2\sigma_1{}^2\sigma_2{}^4-2\rho^2\sigma_1{}^4\sigma_2{}^2+2\rho^3\sigma_1{}^3\sigma_2{}^3)/(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2)^2$$

$$= \sigma_1{}^2\sigma_2{}^2\ [\sigma_2{}^2-2\rho\sigma_1\sigma_2+\rho^2\sigma_1{}^2+\sigma_1{}^2-2\rho\sigma_1\sigma_2+\rho^2\sigma_2{}^2+2\rho\sigma_1\sigma_2 -$$

$$2\rho^2\sigma_2{}^2-2\rho^2\sigma_1{}^2+2\rho^3\sigma_1\sigma_2]/(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2)^2$$

$$= \frac{\sigma_1{}^2\sigma_2{}^2\ (\sigma_1{}^2+\sigma_2{}^2 - 2\rho\sigma_1\sigma_2)(1-\rho^2)}{(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2)^2}$$

$$= \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2} \quad . \tag{19}$$

It is not difficult to show that the error variance of the combined forecast is less than or equal to the error variances of the individual methods. That is, if σ_1^2 is the smallest forecast error variance, then

$$V[e_{T+\tau}(T)] \leq \min\ (\sigma_1^2,\sigma_2^2) = \sigma_1^2$$

$$\frac{\sigma_1^2\sigma_2^2\ (1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\,\rho\,\sigma_1\sigma_2} \leq \sigma_1^2$$

$$\sigma_2^2(1-\rho^2) \leq \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$

$$0 \leq (\sigma_1-\rho\sigma_2)^2 \tag{20}$$

Since the right hand side of this inequality is always nonnegative, we see that the error variance of the combined forecast is always less than or equal to the individual forecast error variances. This result also hold when $\rho=0$. However, when $p=\sigma_1/\sigma_2$, then the right hand side of (20) is zero, implying that one cannot improve the forecasting performance of method 1 by combining it with method 2.

It is also of interest to examine the effect of the correlation coefficient ρ on the combination of two forecasting methods. Suppose that method 1 produces the smallest forecast error variance $\sigma_1^{\,2}$, and let the variance of forecast errors from method 2 be expressed as a multiple of method 1, say $\sigma_2^2 = k\ \sigma_1^{\,2}$, where $k \geq 1$. Then

$$V[e_{T+\tau}(T)] = \frac{\sigma_1^{\,2}\sigma_2^{\,2}(1-\rho^2)}{\sigma_1^{\,2}+\sigma_2^{\,2}-2\rho\sigma_1\sigma_2}$$

$$= \frac{k\ \sigma_1^{\,4}(1-\rho^2)}{\sigma_1^{\,2}(1+k)-2\rho\sigma_1^{\,2}\sqrt{k}}$$

$$= \frac{k\ \sigma_1^{\,2}(1-\rho^2)}{1+k\ -\ 2\rho\sqrt{k}} \tag{21}$$

Note that if ρ=0, then

$$V[e_{T+\tau}(T) = \frac{k\ \sigma_1^{\,2}}{1+k} = \frac{\sigma_1^{\,2}\ \sigma_2^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}\ ,$$

and as noted above, when $\rho=\sigma_1/\sigma_2$,

$$V[e_{T+\tau}(T)] = \sigma_1^{\,2}$$

When $\rho \Rightarrow -1$, we have

$$V[e_{T+\tau}(T) \Rightarrow 0,$$

implying that if two forecasting methods produce forecast errors that are highly negatively correlated, then extremely good results should be expected from the combined forecast. This result has intuitive appeal. Now consider the case where $\rho \Rightarrow +1$. If k=1, we have

$$\lim_{\rho \Rightarrow +1} V[e_{T+\tau}(T)] = \lim_{\rho \Rightarrow +1} \frac{\sigma_1^{\,2}(1-\rho^2)}{2-2\rho} = \sigma_1^{\,2} .$$

That is, if both methods produce forecast errors that are highly positively correlated, and if both methods have the same forecast error variances, then no significant improvement will be obtained by combining them. If k>1, then

$$\lim_{\rho \Rightarrow +1} V[e_{T+\tau}(T)] = \lim_{\rho \Rightarrow +1} \frac{k\,\sigma_1^{\,2}(1-\rho^2)}{1+k-2\rho\sqrt{k}} = 0$$

implying that two forecasting methods with highly positively correlated forecast errors can be combined to produce a single forecast with significantly better forecast error performance.

A graph of $V[e_{T+\tau}(T)]$ versus ρ for various values of k is shown in Figures 3.1 and 3.2. Note that the relationships derived above can be easily seen on these displays. In general, one would expect that forecasting methods that have forecast errors that exhibit high negative correlation can be combined to produce a single forecast with significantly smaller forecast error variance. Also, positively correlated forecast errors indicate that the combined forecast will have a smaller forecast error variance, provided that $\sigma_1^{\,2} \neq \sigma_2^{\,2}$.

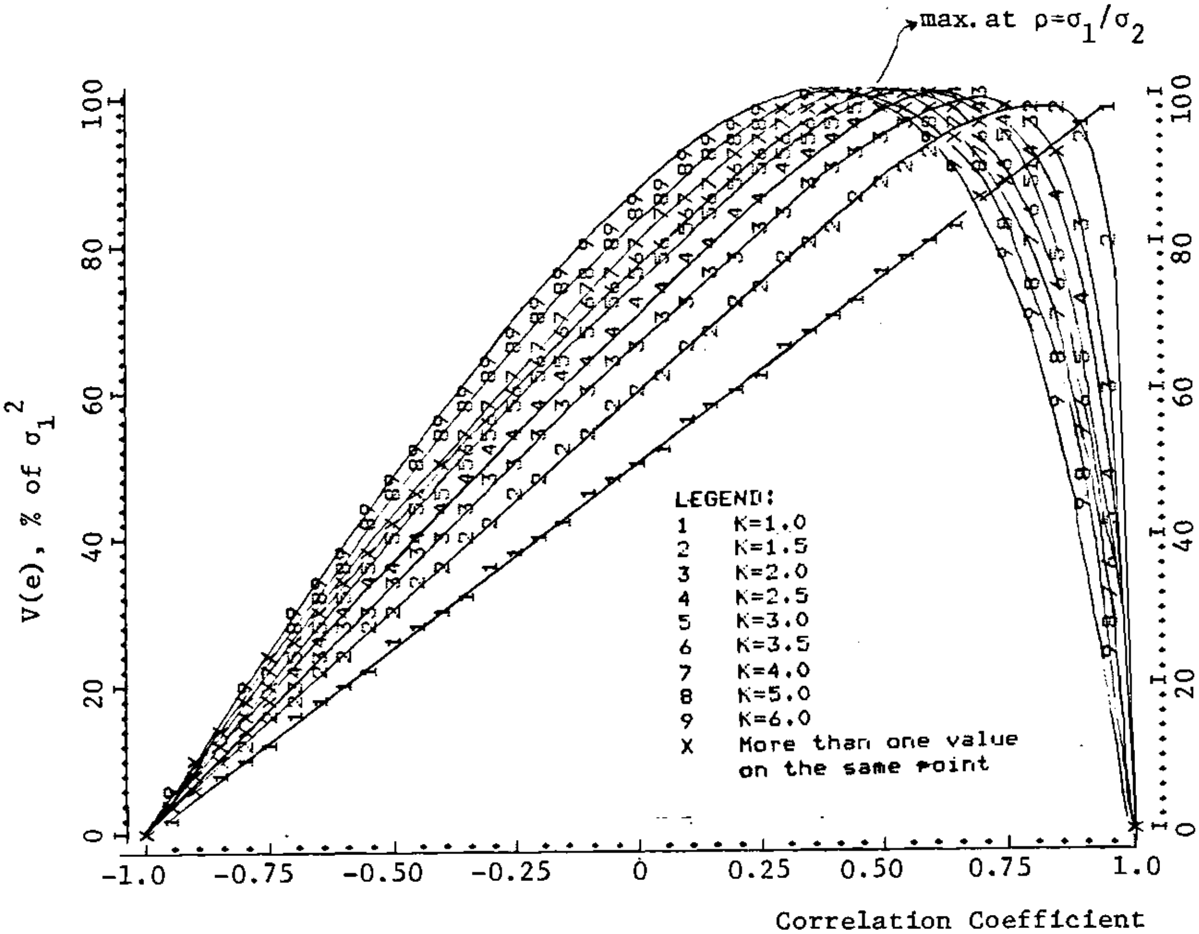


Figure 3.1. Relationship Between Variance of Forecast Error and Correlation Coefficient for the Combination of Two Forecasting Methods (k=1.0 to 6.0)

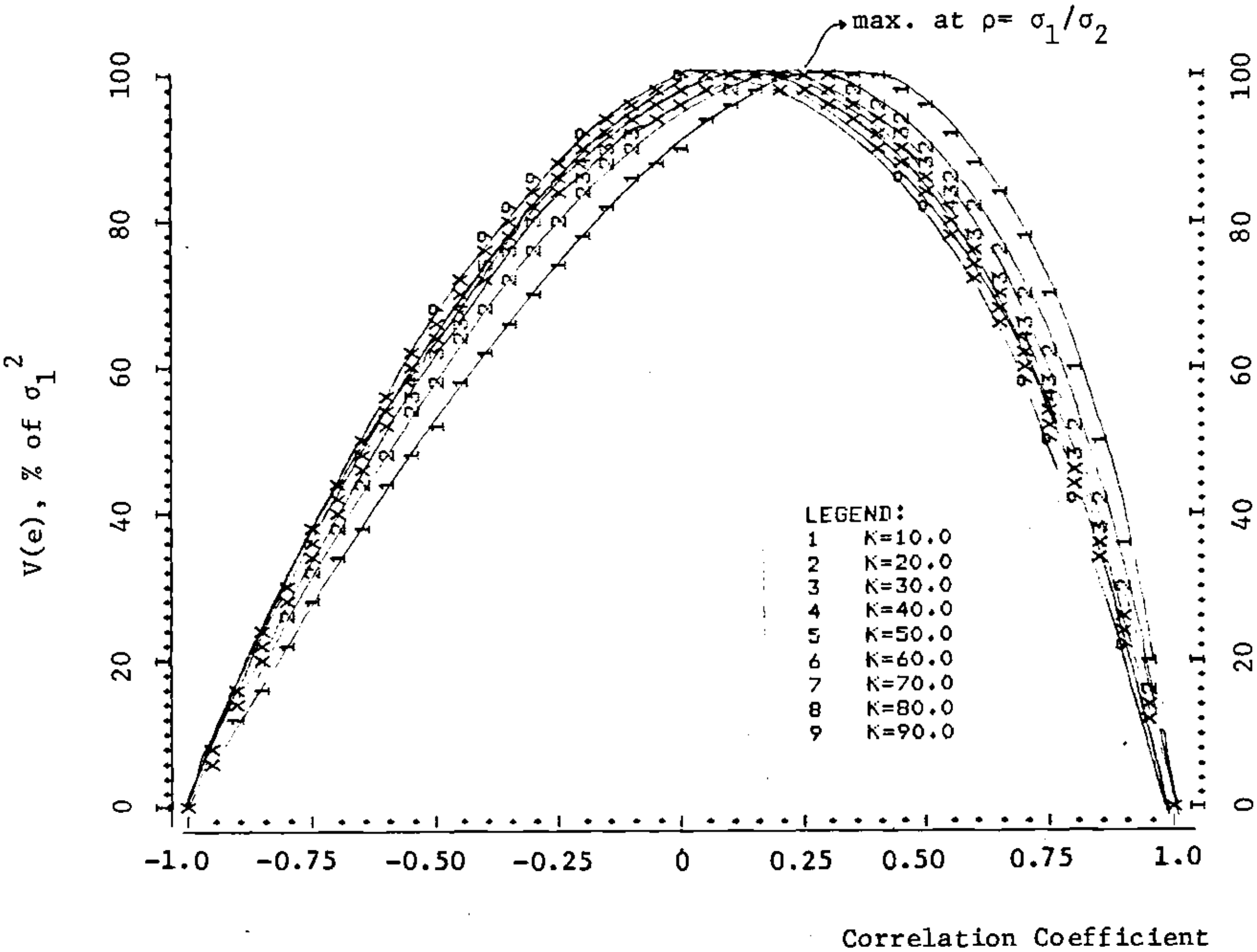


Figure 3.2. Relationship Between Variance of Forecast Error and Correlation Coefficient for the Combination of Two Forecasting Methods (k=10 to 90)

These results can be extended to the case of combining n forecasts. For ease of expression, matrix notation will be used. Using this notation, the combined forecast

$$\hat{X}_{T+\tau}(T) = \sum_{i=1}^{n} W_{i,T}\, \hat{X}_{i,T+\tau}(T) = \underline{W}_T\, \hat{X}_{T+\tau}(T)$$

and the condition $\sum_{i=1}^{n} W_{i,T} = 1$ is written as

$$\underline{1}'\ \underline{W}_T = 1 \tag{22}$$

The forecast error for the combination becomes

$$e_{T+\tau}(T) = \hat{X}_{T+\tau} - \underline{W}_T\, X_{T+\tau}(T) = \underline{W}_T\, [\underline{X}_{T+\tau} - \hat{X}_{T+\tau}(T)]$$

and the variance of the combined forecast

$$V[e_{T+\tau}(T)] = \underline{W}_T'\, V[\underline{X}_{T+\tau} - X_{T+\tau}(T)]\ \underline{W}_T$$

$$= \underline{W}_T'\ \Sigma_{T+\tau}\, \underline{W}_T \tag{23}$$

where $W_T = [W_1, W_2, \quad \ldots \quad , W_n]$

Our objective is to find an optimum method to choose W_T' to minimize $V[e_{T+\tau}(T)]$ subject to $\underline{1}'\underline{W}_T = 1$. Thus by least squares

$$L(\underline{W}_T, \lambda) = \underline{W}_T' \, \Sigma_{T+\tau} \, \underline{W}_T + \lambda(1-\underline{1}'\underline{W}_T)$$

$$\frac{\delta L}{\delta W_T} = 2 \, \Sigma_{T+t} \, \underline{W}_T - \lambda \underline{1} = 0 \tag{24}$$

$$\frac{\delta L}{\delta \lambda} = 1 - \underline{1}' \underline{W}_T = 0$$

implying $\underline{1}' \, \underline{W}_T = 1.$

From (24) we have

$$\underline{W}_T = \frac{1}{2} \, \Sigma_{T+\tau}^{-1} \, \lambda \, \underline{1} \tag{25}$$

and, substituting into (22) gives

$$\left(\frac{1}{2}\lambda\right) \underline{1}' \, \Sigma_{T+\tau}^{-1} \, \underline{1} = 1 \tag{26}$$

But (26) also implies that $\lambda = 2/\underline{1}' \, \Sigma_{T+\tau}^{-1} \underline{1}$, and substituting into (25) gives

$$\underline{W}_T = \frac{1}{2} \, \Sigma_{T+\tau}^{-1} \, (2/\underline{1}' \, \Sigma_{T+\tau} \, \underline{1}) \, \underline{1}$$

$$= \frac{\Sigma_{T+\tau}^{-1} \, \underline{1}}{(\underline{1}' \, \Sigma_{T+\tau}^{-1} \, \underline{1})} \tag{27}$$

as the optimal weights. This result was given by Dickinson [16].

3.4 Weight Updating

The determination of weights is fundamental for the combination of forecasts. Previous sections suggest the use of the covariance matrix ($\Sigma_{T+\tau}$) to estimate the weights. It was found that the variance of the combined forecast errors can be minimized if weights are defined as

$$\underline{W}_T = \frac{\Sigma^{-1}_{T+\tau}\ \underline{1}}{(\underline{1}'\ \Sigma^{-1}_{T+\tau}\ \underline{1})}$$

Since the weights are related to the forecast errors, the better the individual forecasting method, the larger the weight it will receive. It is desirable to have the weights in the combination procedure changing, according to the behavior of the individual method, so that if there is an increase in the magnitude of errors for one specific method, the weight given should decrease. We will calculate the weights each period directly from equation (27). The elements of the covariance matrix at time T will be updated to time T+1 by the use of a smoothing procedure. Recall that the elements of this matrix were defined at the beginning of the chapter as

$$\sum_T = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 & \cdots & \sigma_{1n}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 & \cdots & \sigma_{2n}^2 \\ \sigma_{31}^2 & \sigma_{32}^2 & \sigma_{33}^2 & \cdots & \sigma_{3n}^2 \\ \vdots & \vdots & \vdots & & \vdots \\ \sigma_{n1}^2 & \sigma_{n2}^2 & \sigma_{n3}^2 & \cdots & \sigma_{nn}^2 \end{bmatrix}$$

Specifically, we would like to update the elements of the covariance matrix through the use of some function of the current forecast error, say,

$$\sum_{T+1} = f(e_{T+1}(T), \sum_T).$$

To do this, the following smoothing procedure is proposed. If σ_{ii}^2 is the iith element of $\sum_T$, that is, the variance of forecast errors for method i, then

$$\hat{\sigma}_{ii}^2 (t+1) = \alpha\, e_{i,1}^2(t) + (1 - \alpha)\, \hat{\sigma}_{ii}^2(t) \tag{28}$$

and if σ_{ij}^2 is the ijth element of $\sum_T$, then

$$\hat{\sigma}_{ij}^{2}(t+1) = \alpha[e_{i,t+1}(t)\ e_{j,t+1}(t)] + (1 - \alpha)\ \hat{\sigma}_{ij}^{2}\ (t) \tag{28}$$

where α is a smoothing constant such that $0 < \alpha < 1$. The selection of the appropriate value for α will be discussed later. Our proposed weight updating procedure is to calculate a new covariance matrix $\sum_{T+1}$ by Equations (28) and (29), and to revise the weights for use in forecasting at the end of period T by Equation (27). This weight estimation and updating procedure differs from those previously proposed in the literature in that it smoothes the covariances directly and calculates the weights from least squares optimal formulas rather than smoothing the weights directly in a heuristic manner.

CHAPTER IV

SIMULATION EXPERIMENT

4.1 Introduction

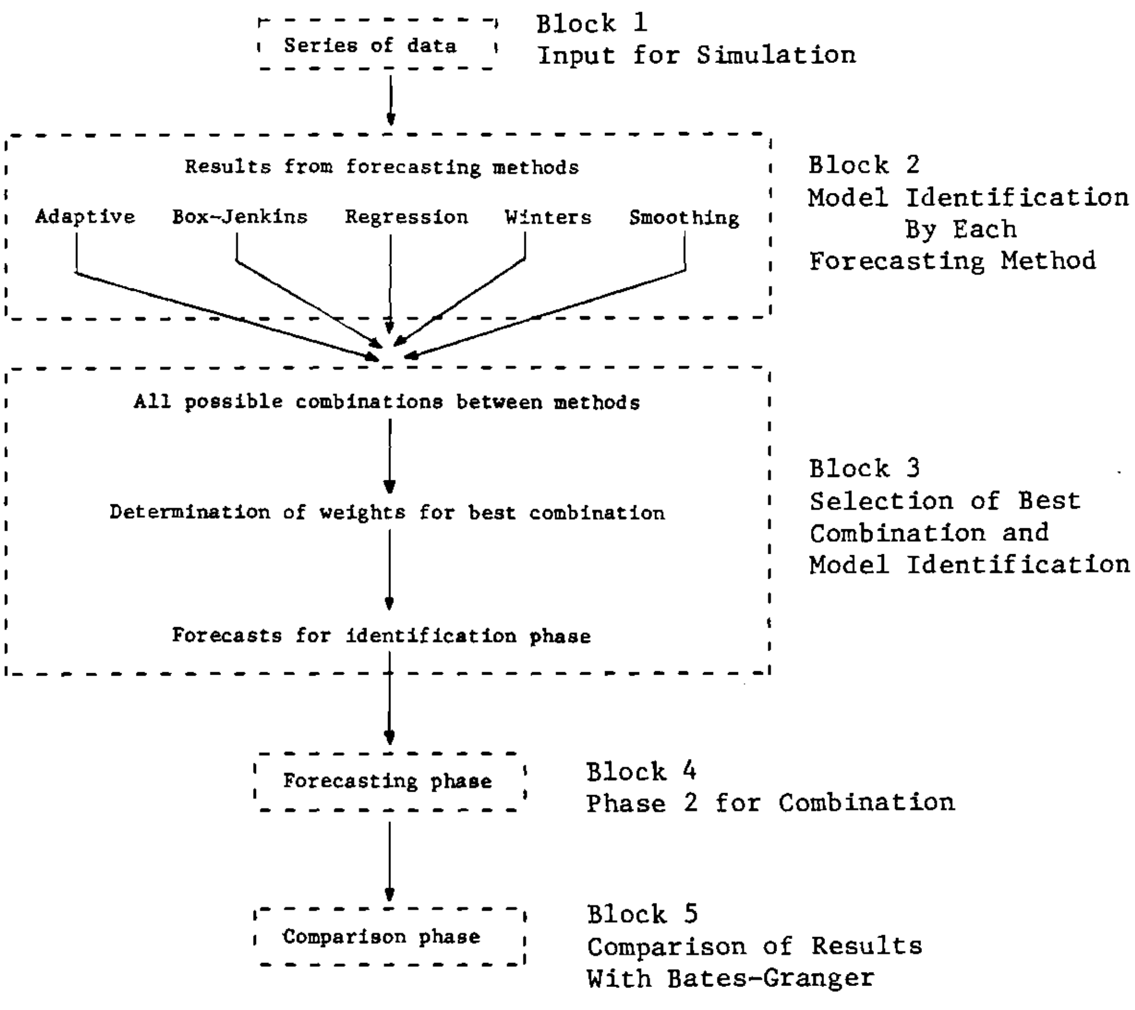
A simulation experiment was performed to study the behavior of the forecast combination method discussed in the previous chapter. This simulation study also compared the performance of the proposed method with others previously suggested in the literature. Twenty actual time series were used in the analysis.

This chapter includes a description of the series and the types of forecasting method for each that seems appropriate: adaptive filtering, Box & Jenkins, linear regression, multiple exponential smoothing, and Winters' Method. Each series was divided into two parts. The first part was used for identification or modeling purposes, and the second part for forecasting.

4.2 Experimental Technique

Several computer programs were used for this part of the analysis. A computer program was written for Adaptive Filtering. A listing of the program and instructions for its use are included on Appendix A. The University of Wisconsin's Box-Jenkins programs [19] were used in the study. A computer program was written to forecast the Box-Jenkins models for lead times other than one. Appendix B contains this program. The linear regression subroutines of the SPSS [20] were used for the regression models. Modified versions of the multiple smoothing and Winters' method programs in Montgomery and Johnson [1] were also used. The combination of forecasts was obtained using program COMB in Appendix C. Detailed instructions on the use of this program, and examples of results are also shown.

The experimental procedure used can be represented by the following diagram



The following sections will discuss this diagram in more detail.

4.3 Time Series Used in the Study

Table 4.1 includes a listing of the series used for the simulation study and listings of the data points are available in Appendix D. Figures 4.1 to 4.20 plot the time series. Tables 4.2 to 4.21 show the results of modeling each time series with the five forecasting techniques used in the research. These results are for the fitting phase only.

For seasonal data, the length of the season for Winters' Method was selected by inspection. For series that do not have an obvious seasonal pattern, Winters' Method with a season (L) equal to one was used. This is really an exponential smoothing with a trend correction with two different rates of smoothing.

Table 4.1. Collection of Time Series

	Series	NOB	NO	Figure
1.	International Airline Passengers-Monthly totals - Jan 1949-Dec 1960	144	80	4.1
2.	IBM Common Stock Closing Prices-Daily, 17th May 1961-2nd Nov 1962	369	180	4.2
3.	IBM Common Stock Closing Prices-Daily, 29th June 1959-30th June 1960	255	148	4.3
4.	Chemical Process Viscosity Readings - Every Hour	310	160	4.4
5.	U.S. Auto Registrations in Thous. Monthly 1947-01, 1968-12	264	132	4.5
6.	Chemical Process Temperature Readings - Every Two Minutes	100	75	4.6
7.	Yearly Wolfer Sunspot Numbers (Average numbers of sunspots/year)	100	75	4.7
8.	Minutes of Usage per day of a Computer Terminal	100	75	4.8
9.	Chemical Process Temperature Readings - Every Minute	226	113	4.9
10.	Demand for a Double Knit Polyester Fabric	240	120	4.10
11.	U.S. Treasury Bills Interest Rate, Monthly Jan 1956-Jan 1969	157	85	4.11
12.	Monthly Value of Residential Construction - Jan 1959-Dec 1969	132	66	4.12
13	U.S. Documented Merchant Vessels - Trade Sailing 1789-1970	182	100	4.13
14.	Weekly Sales of a Cutting Tool	100	75	4.14
15.	Dow Jones Transportation	100	75	4.15
16.	Delta Airlines	100	75	4.16
17.	National Airlines	100	75	4.17
18.	Eastern Airlines	100	75	4.18
19.	Chemical Process Concentration Readings - Every Two Hours	197	100	4.19
20.	Monthly Champagne Sales	105	75	4.20

NOB: Total number of observations
NO: Number of observations used in modeling

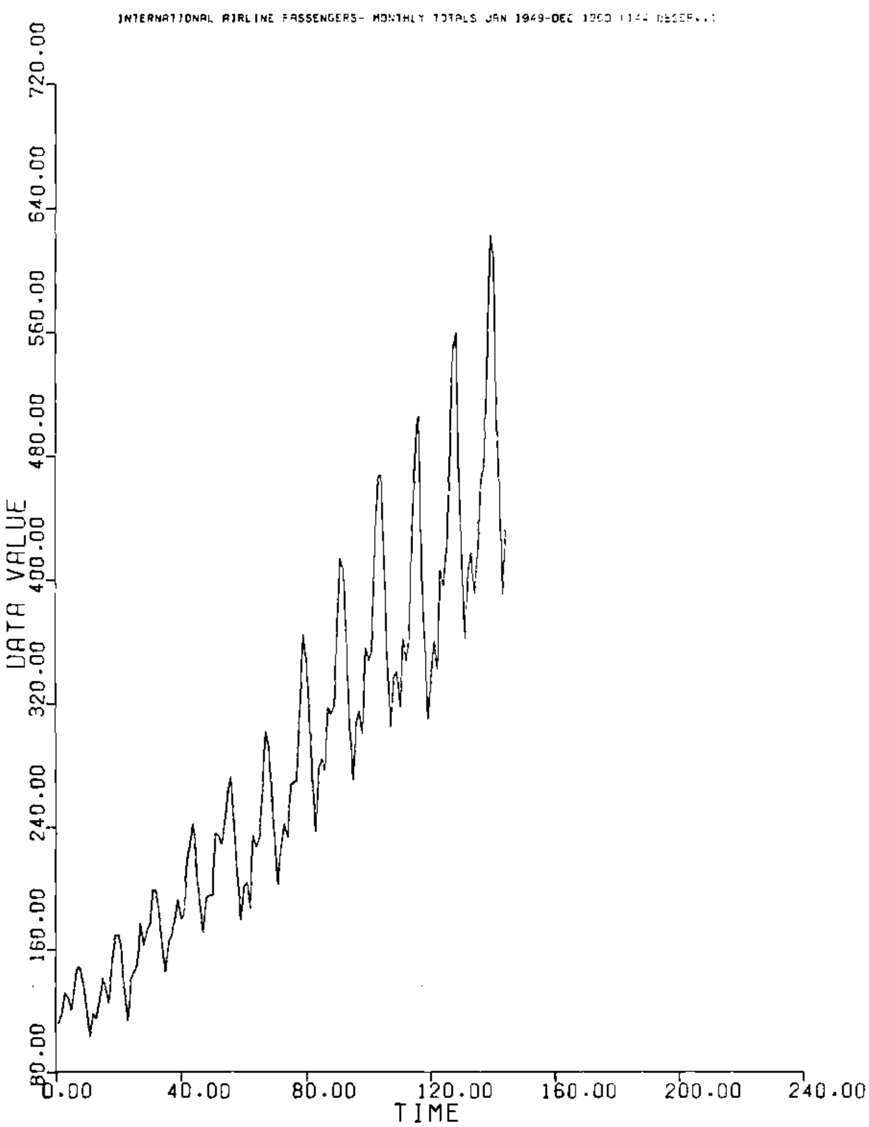


Figure 4.1. Series 1 International Airline Passengers

Table 4.2 Alternative Forecasting Models for Series 1 International Airline Passengers

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.1179$	$p = 0$	Slope = 2.1756	Second Order	$\hat{c}_1 = 0.9060$
$\hat{w}_2 = -0.0281$	$d = 1$	Intercept=105.34	Intercept=133.14	$\hat{c}_2 = 0.9210$
$\hat{w}_3 = 0.0674$	$q = 1$		$\alpha = 0.63$	$\hat{c}_3 = 1.05$
$\hat{w}_4 = -0.0223$	$P = 0$		S1 = 131.1125	$\hat{c}_4 = 0.9990$
$\hat{w}_5 = 0.0066$	$D = 1$		S2 = 129.0821	$\hat{c}_5 = 0.9780$
$\hat{w}_6 = 0.0033$	$Q = 1$			$\hat{c}_6 = 1.08$
$\hat{w}_7 = 0.0157$	$S = 12$			$\hat{c}_7 = 1.19$
$\hat{w}_8 = -0.0585$	$\theta = 0.3569$			$\hat{c}_8 = 1.18$
$\hat{w}_9 = 0.0194$	$\Theta = 0.6719$			$\hat{c}_9 = 1.05$
$\hat{w}_{10} = -0.0894$	Log used			$\hat{c}_{10} = 0.92$
$\hat{w}_{11} = 0.2839$				$\hat{c}_{11} = 0.802$
$\hat{w}_{12} = 0.8580$				$\hat{c}_{12} = 0.910$
$k = 0.35$				$\alpha = 0.7$
80 iterations				$\beta = 0.2$
				$\gamma = 0.2$
				$\hat{b}_1(0) = 117.00$
				$\hat{b}_2(0) = 1.65$
$MSE_I = 101.60$	85.12	718.03	559.79	322.48
$MSE_F = 1350.19$	164.74	5559.50	2799.73	398.07

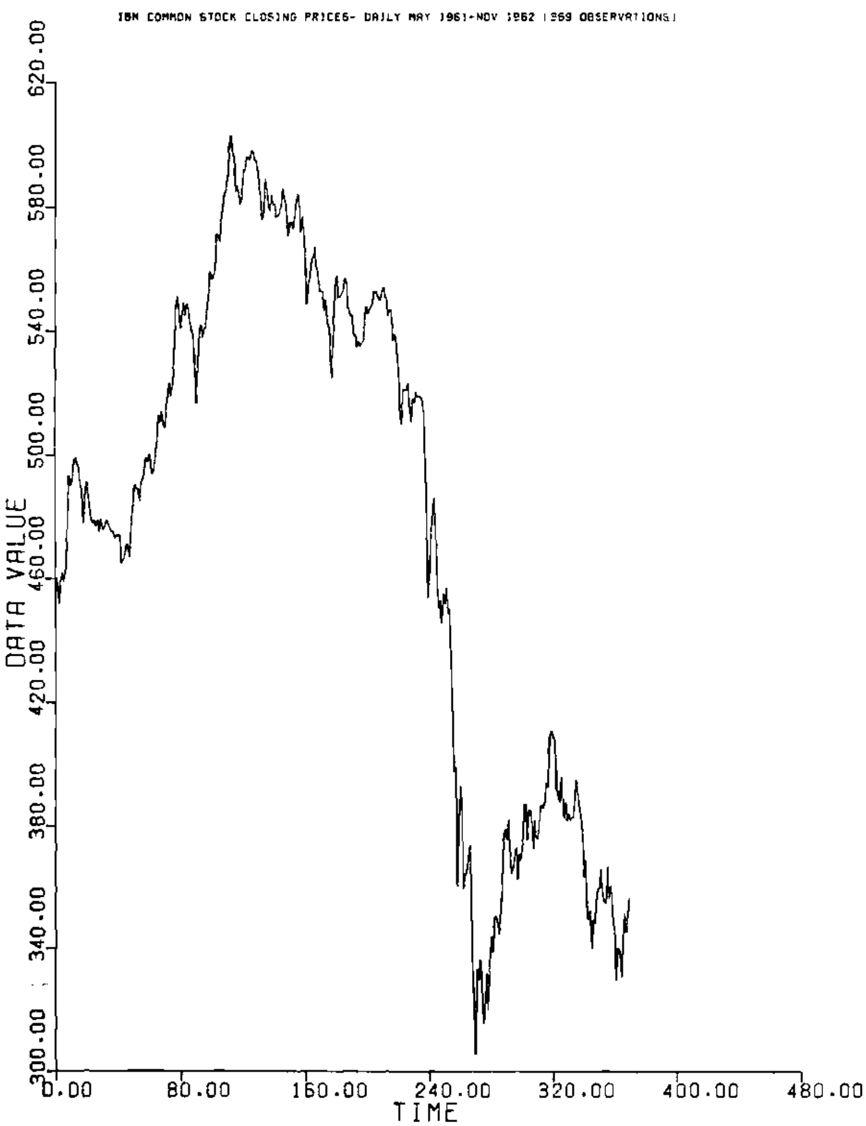


Figure 4.2. Series 2 IBM Common Stock Closing Prices

Table 4.3 Alternative Forecasting Models for Series 2 IBM Common Stock Closing Prices

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters' Method
$\hat{w}_1$ = - 0.0286	IMA(1,1)	Slope=0.71650	Third Order or	$\hat{c}_1$=1.0184
$\hat{w}_2$ = -0.0553	d=1	Intercept=468.37	Quadratic Model	$\hat{c}_2$=1.0039
$\hat{w}_3$ = -0.0169	q=1		α = 0.42	$\hat{c}_3$=1.0057
$\hat{w}_4$ = 0.0794	θ = 0.11184		S1 = 469.11	$\hat{c}_4$=1.0019
$\hat{w}_5$ = 0.0335			S2 = 479.11	$\hat{c}_5$=0.9977
$\hat{w}_6$ = -0.0113			S3 = 491.29	$\hat{c}_6$=0.9966
$\hat{w}_7$ = 0.0233			Intercept=461.29	$\hat{c}_7$=0.9966
$\hat{w}_8$ = 0.0881			Linear comp=-3.51	$\hat{c}_8$=0.9957
$\hat{w}_9$ = 0.1185			Quad. comp=1.14	$\hat{c}_9$=0.9930
$\hat{w}_{10}$ = 0.1814				$\hat{c}_{10}$=0.9938
$\hat{w}_{11}$ = 0.2623				$\hat{c}_{11}$=0.9976
$\hat{w}_{12}$ = 0.3353				$\hat{c}_{12}$=0.9990
k = 0.60				α=0.7
				β=0.2
				γ=0.2
				$\hat{b}_1$(0)=557.4604
				$\hat{b}_2$(0)=2.4513
MSE_I= 38.327	30.847	584.487	45.240	107.873
MSE_F=1091.490	77.681	70754.6	112.668	102.166

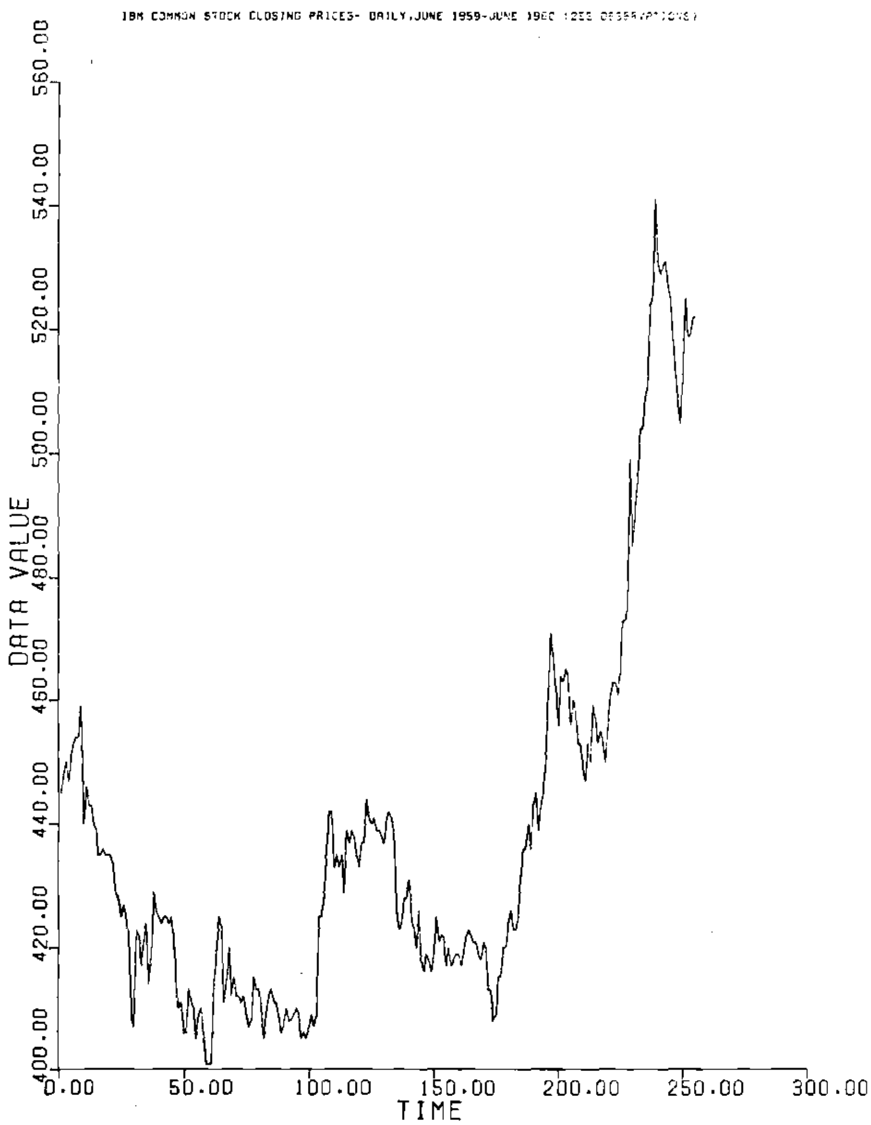


Figure 4.3. Series 3 IBM Common Stock Closing Prices

Table 4.4 Alternative Forecasting Models for Series 3 - IBM Common Stock Closing Prices: Daily, 29th June 1959-30th June 1960

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters' Method
$\hat{w}_1$ = .0613	IMA(1,1)	Slope:-.03630	Third Order	$\hat{c}_1$=0.9951
$\hat{w}_2$ = .0474	d=1	Intercept=426.556	Intercept:452.46	$\hat{c}_2$=0.9985
$\hat{w}_3$ = .0696	q=1		Linear comp=0.94	$\hat{c}_3$=0.9995
$\hat{w}_4$ = .0718	Θ=0.11184		Quad comp=0.11	$\hat{c}_4$=1.0015
$\hat{w}_5$ = .0736			S1 = 450.81	$\hat{c}_5$=1.0002
$\hat{w}_6$ = .0961			S2 = 450.03	$\hat{c}_6$=1.0014
$\hat{w}_7$ = .0899			S3 = 450.11	$\hat{c}_7$=1.0033
$\hat{w}_8$ = .0857			α = 0.26	α=0.70
$\hat{w}_9$ = .0904				β=0.20
$\hat{w}_{10}$ = .1172				γ=0.20
$\hat{w}_{11}$ = .1162				$\hat{b}_1(0)$=450.4271
$\hat{w}_{12}$ = .1105				$\hat{b}_2(0)$=-0.2036
k = 0.60				
MSE_I = 22.145	22.286	193.842	30.393	24.465
MSE_F =409.782	30.263	3112.847	36.149	33.611

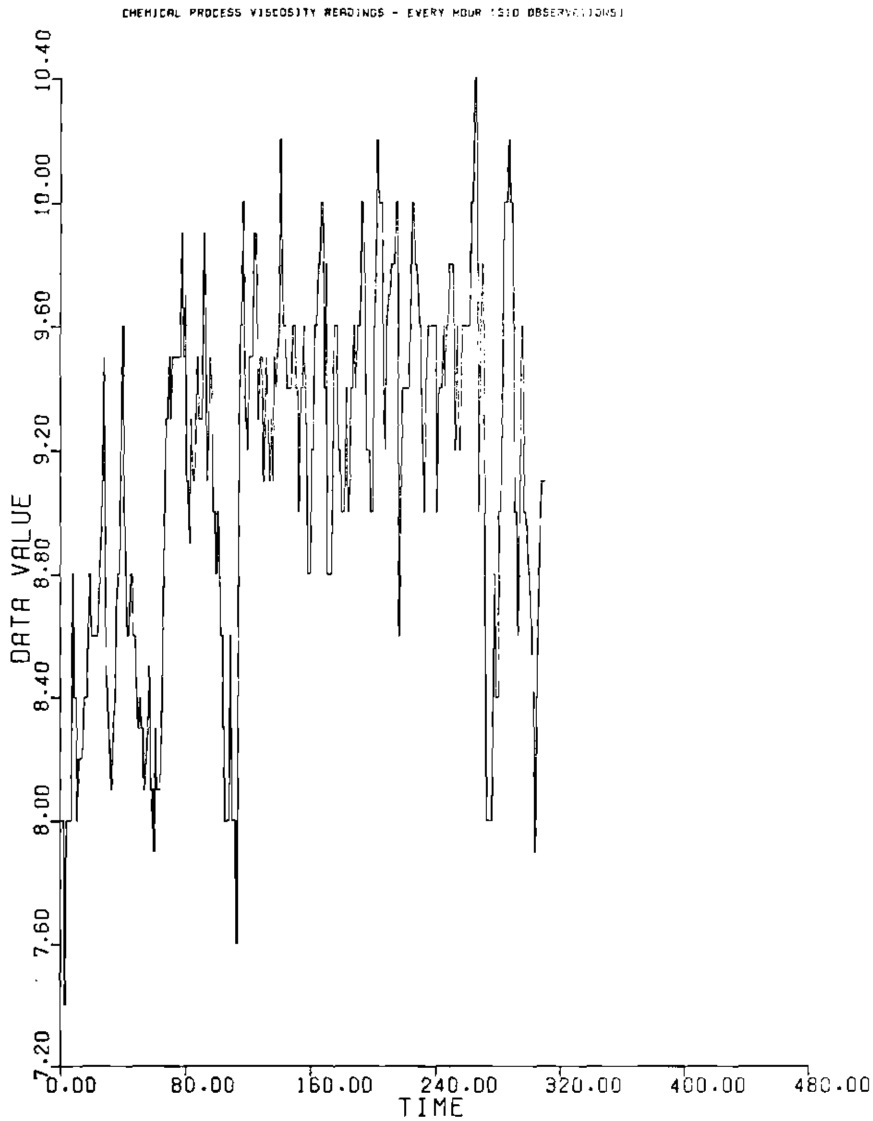


Figure 4.4. Series 4 Chemical Process Viscosity Readings

Table 4.5 Alternative Forecasting Models for Series 4 Chemical Process Viscosity Readings

	Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
	$\hat{w}_1$ = 0.9985	IMA(1,1)	Slope:0.00766	First Order	$\hat{c}_1(0)$ = 1.0024
	k = 0.02	d=1	Intercept:8.29	Intercept=7.9	$\hat{c}_2(0)$ = 0.9976
		q=1		α=0.80	α = 0.7
		θ=0.13852		S1=7.90	β = 0.2
					γ = 0.2
					$\hat{b}_1(0)$ = 7.9949
					$\hat{b}_2(0)$ = 0.0051
MSE_I =	0.088	0.085	0.231	0.083	0.131
MSE_F =	0.108	0.112	0.975	0.112	0.139

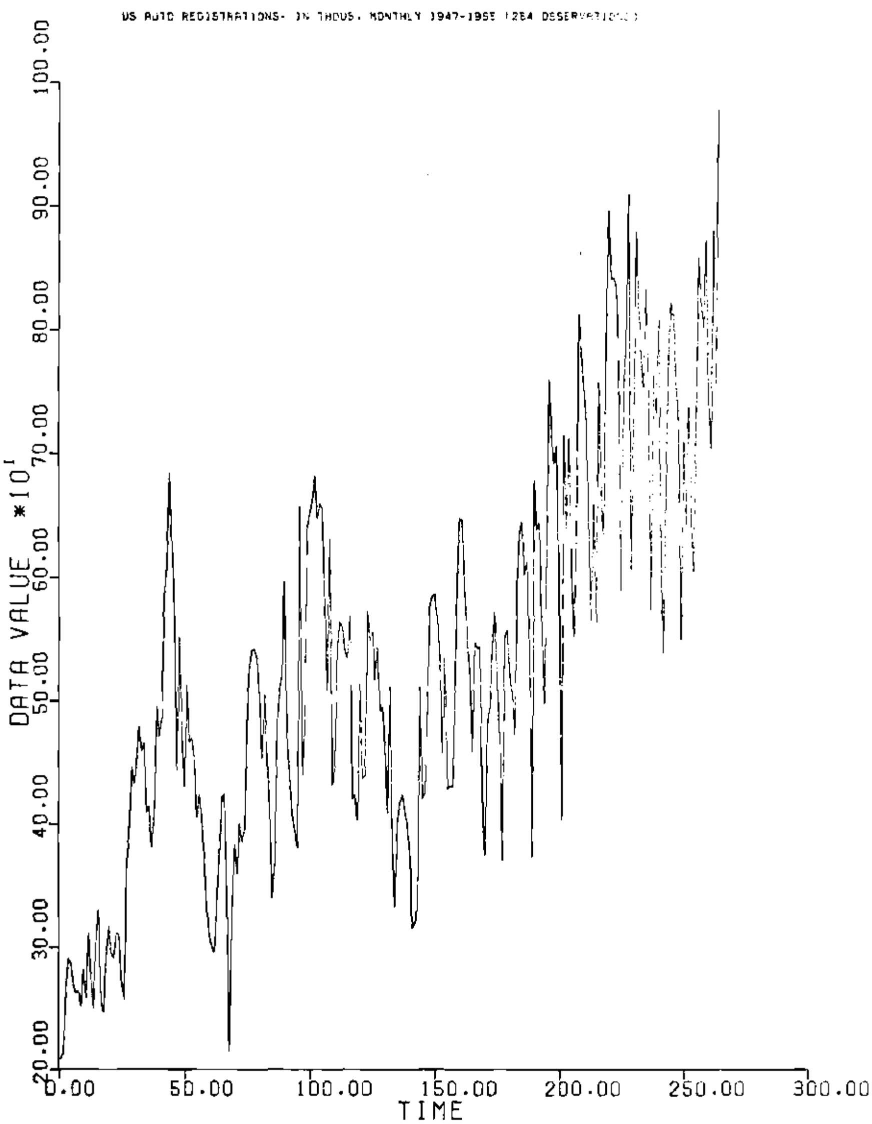


Figure 4.5. Series 5 U.S. Auto Registrations

Table 4.6 Alternative Forecasting Models for Series 5 - U.S. Auto Registrations - in Thous. - Monthly 1947-1958

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1$ = .3468	Multiplicative	Slope =1.92246	Third Order	$\hat{c}_1$=0.9512
$\hat{w}_2$ = -.0527	p=0	Intercept:308.01	Intercept=275.64	$\hat{c}_2$=0.8501
$\hat{w}_3$ = .0523	d=1		Linear comp=56.44	$\hat{c}_3$=1.0382
$\hat{w}_4$ = -.0671	q=2		Quad Comp = -11.64	$\hat{c}_4$=1.0821
$\hat{w}_5$ = -.0407	P=0		S1 = 38.57	$\hat{c}_5$=1.0848
$\hat{w}_6$ = .0230	D=1		S2 = -275.49	$\hat{c}_6$=1.0966
$\hat{w}_7$ = -0.0256	Q=1		S3 = -66.54	$\hat{c}_7$=1.0477
$\hat{w}_8$ = .0542	S=12		α = 0.28	$\hat{c}_8$=1.0330
$\hat{w}_9$ = .0925	Θ_1 = -.38199			$\hat{c}_9$=0.9795
$\hat{w}_{10}$ = -.0171	Θ_2 = -.0052			$\hat{c}_{10}$=0.9676
$\hat{w}_{11}$ = .0281	Θ = .4867			$\hat{c}_{11}$=0.8786
$\hat{w}_{12}$ = .6333	Log used			$\hat{c}_{12}$=0.9906
k = 0.20				α = 1.60
				β = 0.20
				γ = 0.20
				$\hat{b}_1(0)$=252.1708
				$\hat{b}_2(0)$=1.9576
MSE_I = 4267.78	5421.426	8445.74	5631.88	4771.98
MSE_F = 9678.41	4305.282	*	12556.90	4735.83

*Values greater than 10^5 not considered

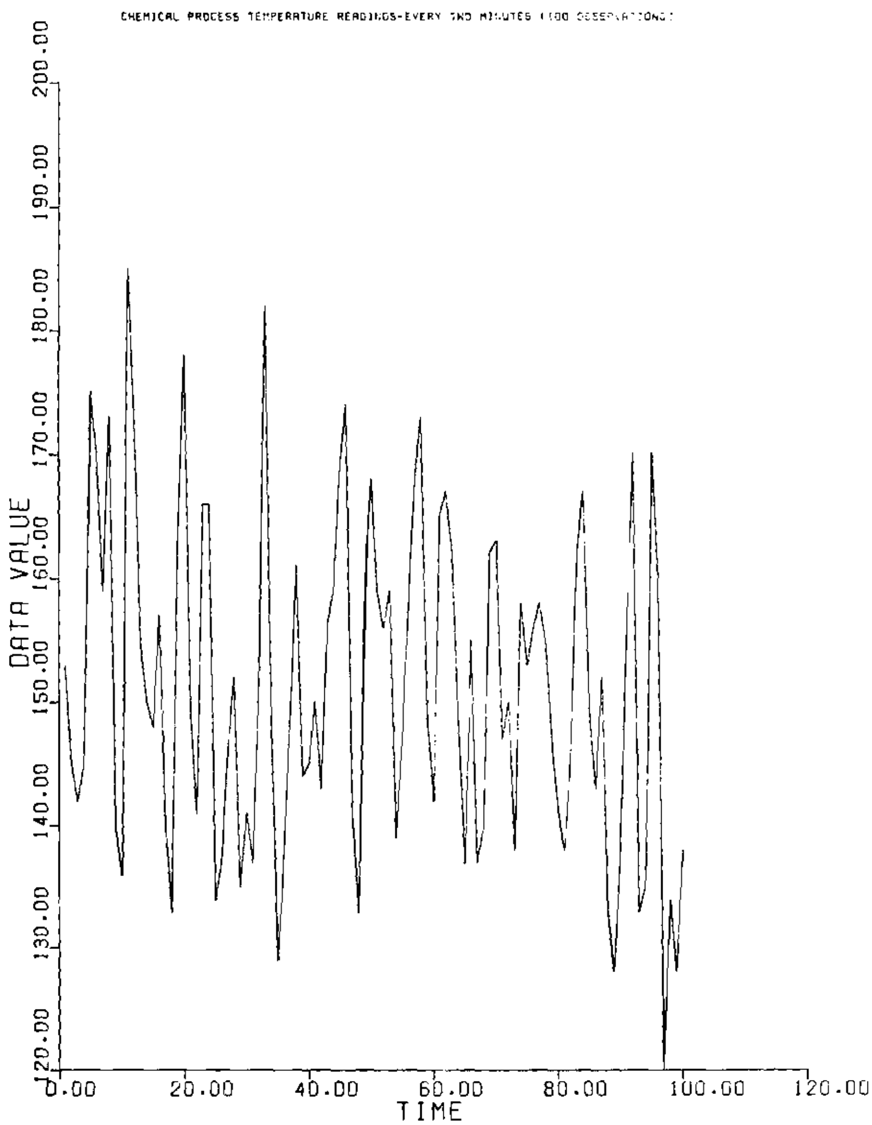


Figure 4.6. Series 6 Chemical Process Temperature Readings

Table 4.7 Alternative Forecasting Methods for Series 6 - Chemical Process Temperature Readings

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.1270$	ARIMA(1,1,0)	Slope=-0.024	First Order	$\hat{c}_1 = 1.00$
$\hat{w}_2 = -0.3180$	p=1	Intercept=153.69	Intercept=155	$\alpha = 0.20$
$\hat{w}_3 = 0.0452$	d=1		S1 = 155.00	$\beta = 0.20$
$\hat{w}_4 = 0.1383$	q=0		$\alpha = 0.10$	$\gamma = 0.20$
$\hat{w}_5 = 0.2389$	ϕ=-0.11676			$\hat{b}_1(0) = 1530$
$\hat{w}_6 = 0.1866$				$\hat{b}_2(0) = 0.0$
$\hat{w}_7 = 0.1724$				
$\hat{w}_8 = 0.2806$				
$\hat{w}_9 = 0.0647$				
$\hat{w}_{10} = 0.0654$				
k = 0.14				
MSE_I = 151.016	271.375	176.145	182.522	78.837
MSE_F = 272.801	290.669	233.128	199.27	233.356

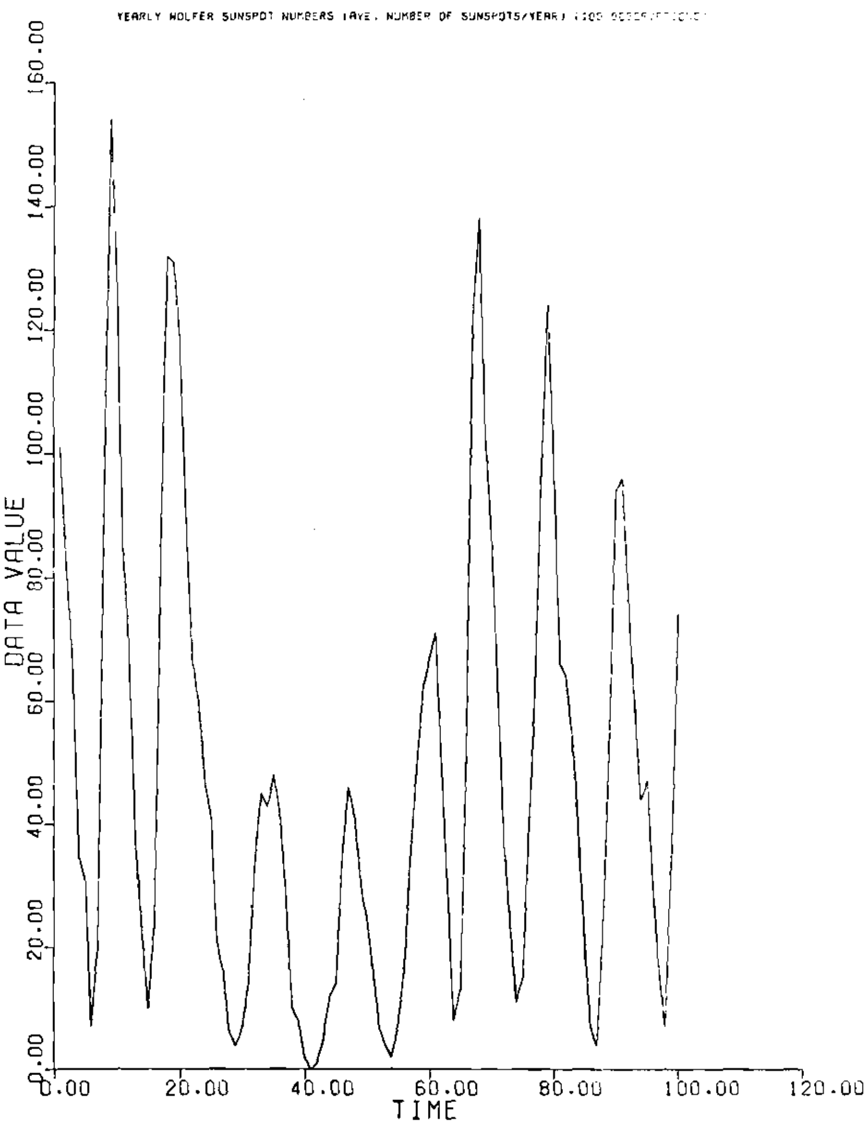


Figure 4.7. Series 7 Yearly Wolfer Sunspot Numbers

Table 4.8 Alternative Forecasting Methods for Series 7 - Yearly Wolfer Sunspot Numbers

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 1.3136$	AR(3)	Slope= -0.3615	Second Order	$\hat{c}_1 = 0.7853$
$\hat{w}_2 = -0.4636$	p=3	Intercept=58.52	Intercept=6.95	$\hat{c}_2 = 0.5927$
$\hat{w}_3 = -0.1675$	$\phi_1 = 1.5569$		Slope = -18.69	$\hat{c}_3 = 0.3875$
$\hat{w}_4 = 0.0723$	$\phi_2 = -1.0103$		S1 = 11.62	$\hat{c}_4 = 0.2168$
$\hat{w}_5 = -0.0179$	$\phi_3 = 0.22546$		S2 = 16.30	$\hat{c}_5 = 0.2801$
$\hat{w}_6 = 0.0516$	Mean = 43.133		$\alpha = 0.80$	$\hat{c}_6 = 0.3290$
$\hat{w}_7 = 0.0445$				$\hat{c}_7 = 0.7885$
$\hat{w}_8 = 0.0239$				$\hat{c}_8 = 1.3731$
$\hat{w}_9 = 0.1245$				$\hat{c}_9 = 1.7324$
$\hat{w}_{10} = -0.0012$				$\hat{c}_{10} = 1.7290$
$\hat{w}_{11} = 0.0061$				$\hat{c}_{11} = 1.5119$
$\hat{w}_{12} = 0.1102$				$\hat{c}_{12} = 1.0807$
$k = 0.06$				$\hat{c}_{13} = 0.5364$
				$\hat{c}_{14} = 0.2922$
				$\hat{c}_{15} = 0.1779$
				$\hat{c}_{16} = 0.5525$
				$\hat{c}_{17} = 1.3651$
				$\hat{c}_{18} = 1.8130$
				$\hat{c}_{19} = 1.6735$
				$\hat{c}_{20} = 1.5299$
				$\hat{c}_{21} = 1.6154$
				$\hat{c}_{22} = 1.5263$
				$\hat{c}_{23} = 1.3171$
				$\hat{c}_{24} = 0.9466$
				$\hat{c}_{25} = 0.7631$
				$\alpha = 0.7$
				$\beta = 0.2$
				$\gamma = 0.2$
$MSE_I = 194.573$	244.753	1468.413	553.93	5158.23
$MSE_F = 397.43$	225.575	1788.25	481.63	15238.7

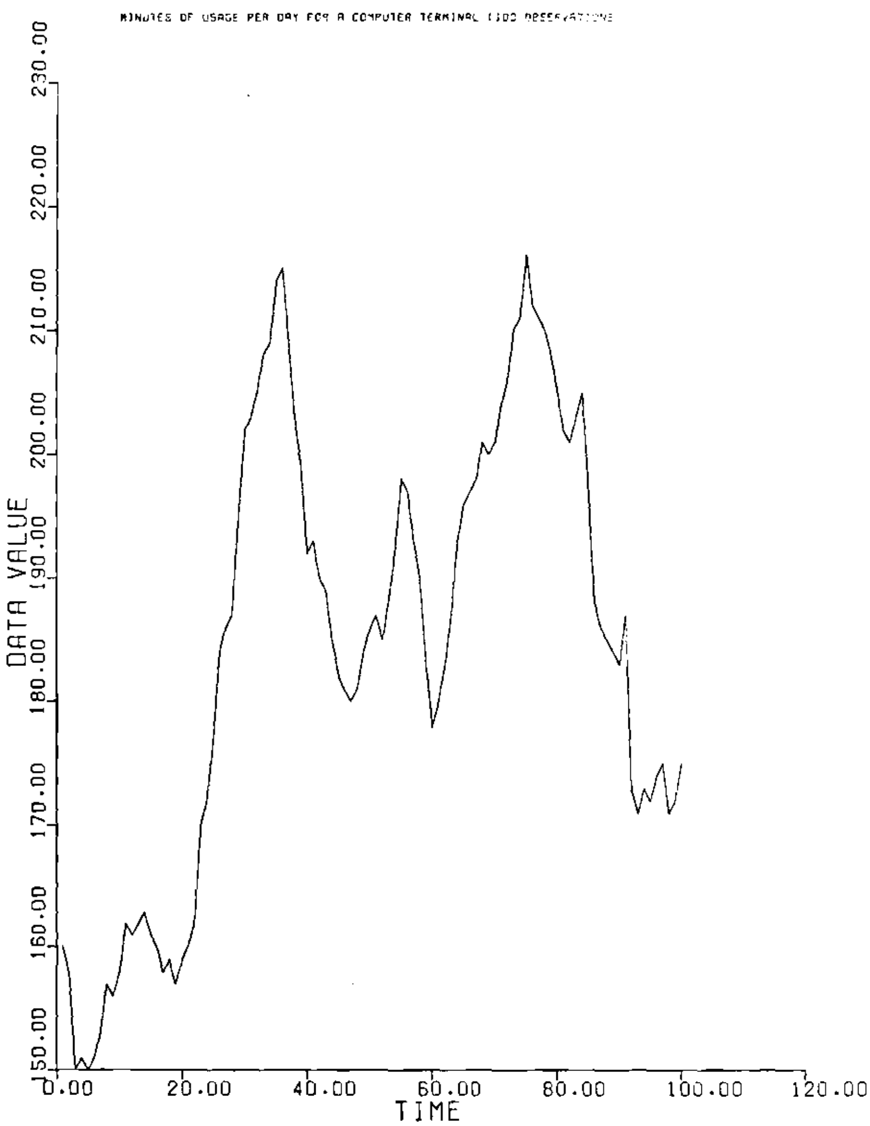


Figure 4.8. Series 8 Minutes of Usage Per Day of a Computer Terminal

Table 4.9 Alternative Forecasting Methods for Series 8 - Minutes of Usage per day for a Computer Terminal

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 1.0167$	ARIMA(2,1,0)	Slope=0.8378	First Order	$\hat{c}_1 = 1.0$
$k = 0.25$	$p=2$	Intercept=149.02	Intercept=153.33	$\alpha = 0.70$
	$d=1$		S1 = 153.33	$\beta = 0.50$
	$\phi_1=0.44142$		$\alpha = 0.30$	$\gamma = 0.70$
	$\phi_2=0.12476$			$\hat{b}_1(0)=159.62$
				$\hat{b}_2(0)=0.7568$
$MSE_I = 10.594$	8.934	181.056	56.153	7.383
$MSE_F = 39.266$	23.073	1637.830	45.454	21.855

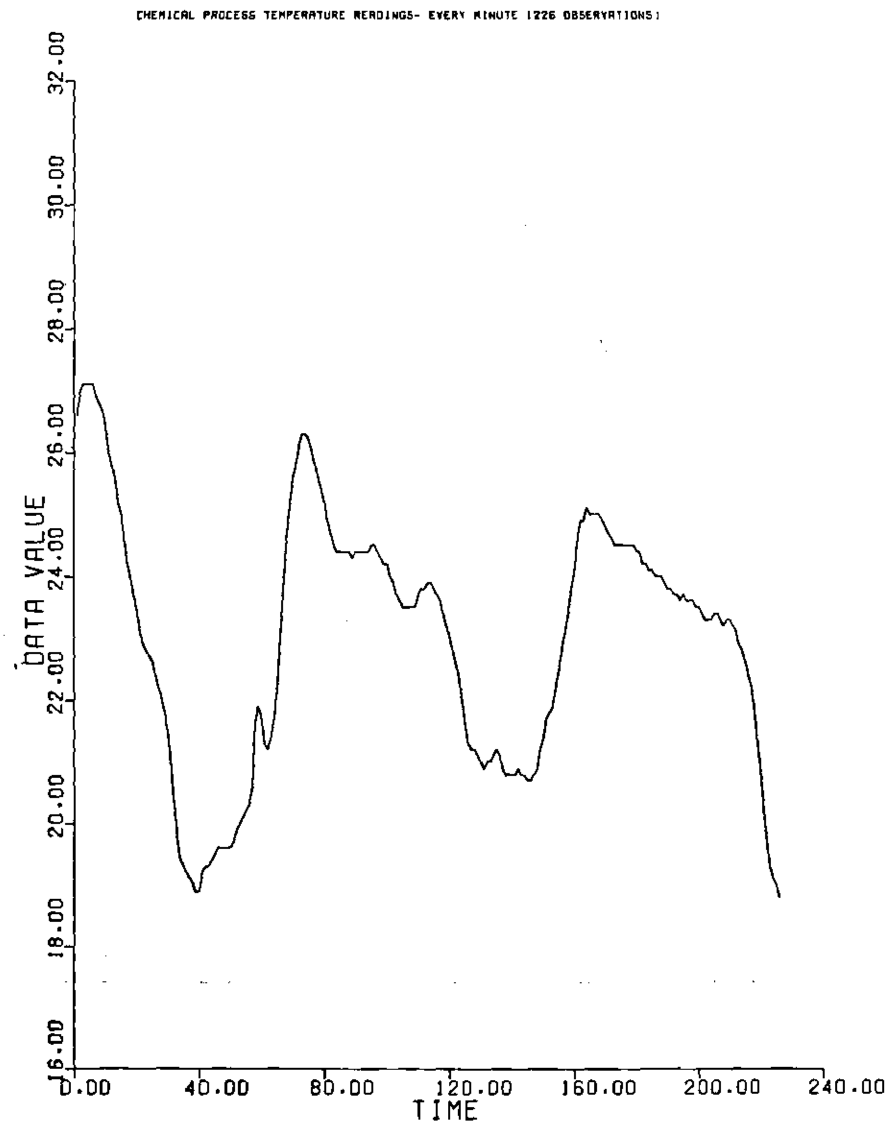


Figure 4.9. Series 9 Chemical Process Temperature Readings

Table 4.10 Alternative Forecasting Methods for Series 9 - Chemical Process Temperature Readings - Every Minute

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.3800$	ARIMA(1,1,0)	Slope=0.0049	First Order	$\hat{c}_1 = 1.0$
$\hat{w}_2 = 0.6268$	p=1	Intercept=22.91	Intercept=27.00	$\alpha = 0.70$
k = 0.0284	d=1		Slope=-31.11	$\beta = 0.70$
	ϕ=0.80444		S1 = 27.00	$\gamma = 0.70$
			$\alpha = 0.30$	$\hat{b}_1(0) = 26.61$
				$\hat{b}_2(0) = -0.0241$
$MSE_I = 0.028$	0.025	5.965	0.515	0.013
$MSE_F = 0.127$	0.012	3.474	0.283	0.012

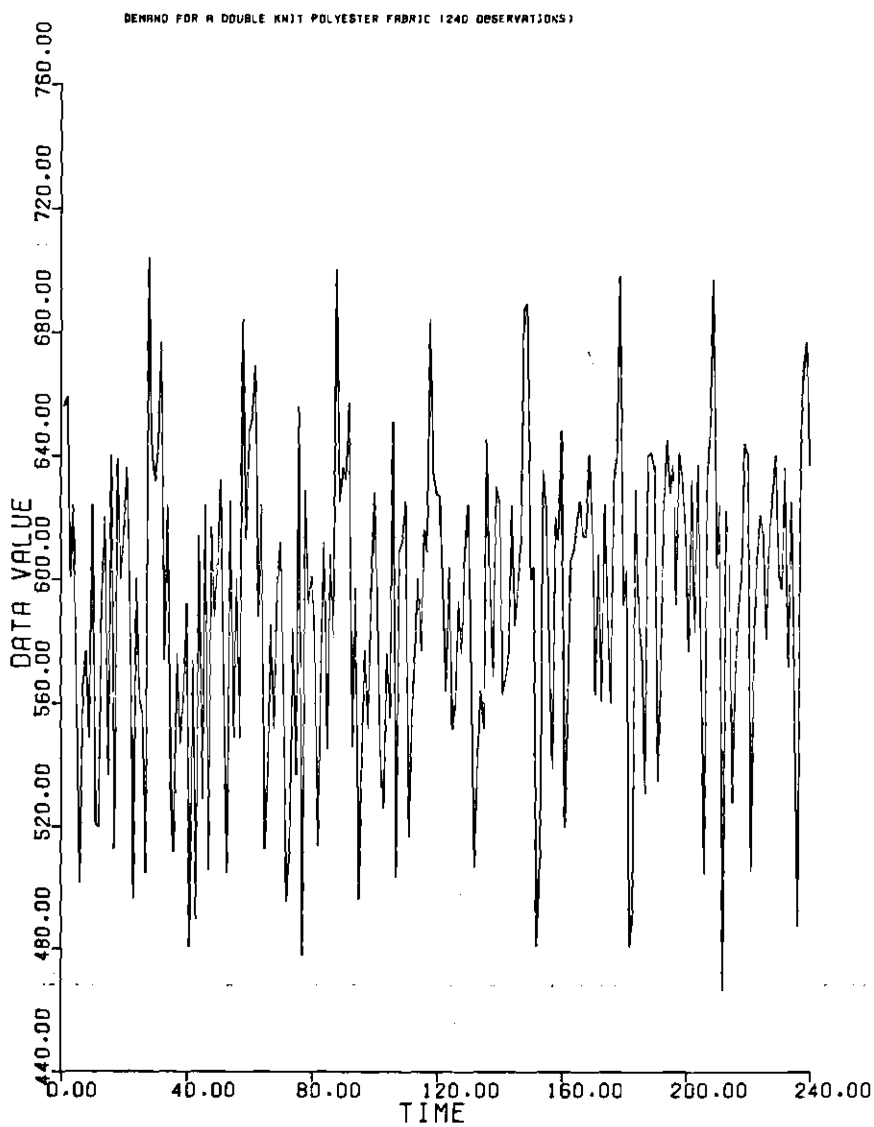


Figure 4.10. Series 10 Demand for a Double Knit Polyester Fabric

Table 4.11 Alternative Forecasting Methods for Series 10 - Demand for a Double Knit Polyester Fabric

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.0233$	MA(2)	Slope=0.0568	Second Order	$\hat{c}_1 = 1.0062$
$\hat{w}_2 = 0.1827$	p=0	Intercept=581.80	Intercept=520.05	$\hat{c}_2 = 1.0319$
$\hat{w}_3 = 0.0030$	d=0		Slope=-31.11	$\hat{c}_3 = 0.9678$
$\hat{w}_4 = 0.0702$	q=2		S1 = 630.36	$\hat{c}_4 = 1.0735$
$\hat{w}_5 = -0.0124$	$\Theta_1 = -0.0535$		S2 = 740.68	$\hat{c}_5 = 0.9348$
$\hat{w}_6 = 0.0280$	$\Theta_2 = -0.27087$		$\alpha = 0.22$	$\hat{c}_6 = 1.0102$
$\hat{w}_7 = -0.0321$	Mean = 585.97			$\hat{c}_7 = 0.9834$
$\hat{w}_8 = 0.1593$				$\hat{c}_8 = 1.0394$
$\hat{w}_9 = 0.1337$				$\hat{c}_9 = 0.9774$
$\hat{w}_{10} = 0.1821$				$\hat{c}_{10} = 1.0607$
$\hat{w}_{11} = 0.0449$				$\hat{c}_{11} = 0.9275$
$\hat{w}_{12} = 0.2699$				$\hat{c}_{12} = 0.9872$
$k = 0.10$				$\alpha = 0.5$
				$\beta = 0.2$
				$\gamma = 0.2$
				$\hat{b}_1(0) = 577.23$
				$\hat{b}_2(0) = 0.2253$
$MSE_I = 2550.82$	2467.07	2698.31	3494.68	3376.14
$MSE_F = 3753.73$	1214.18	2173.17	2863.98	3348.01

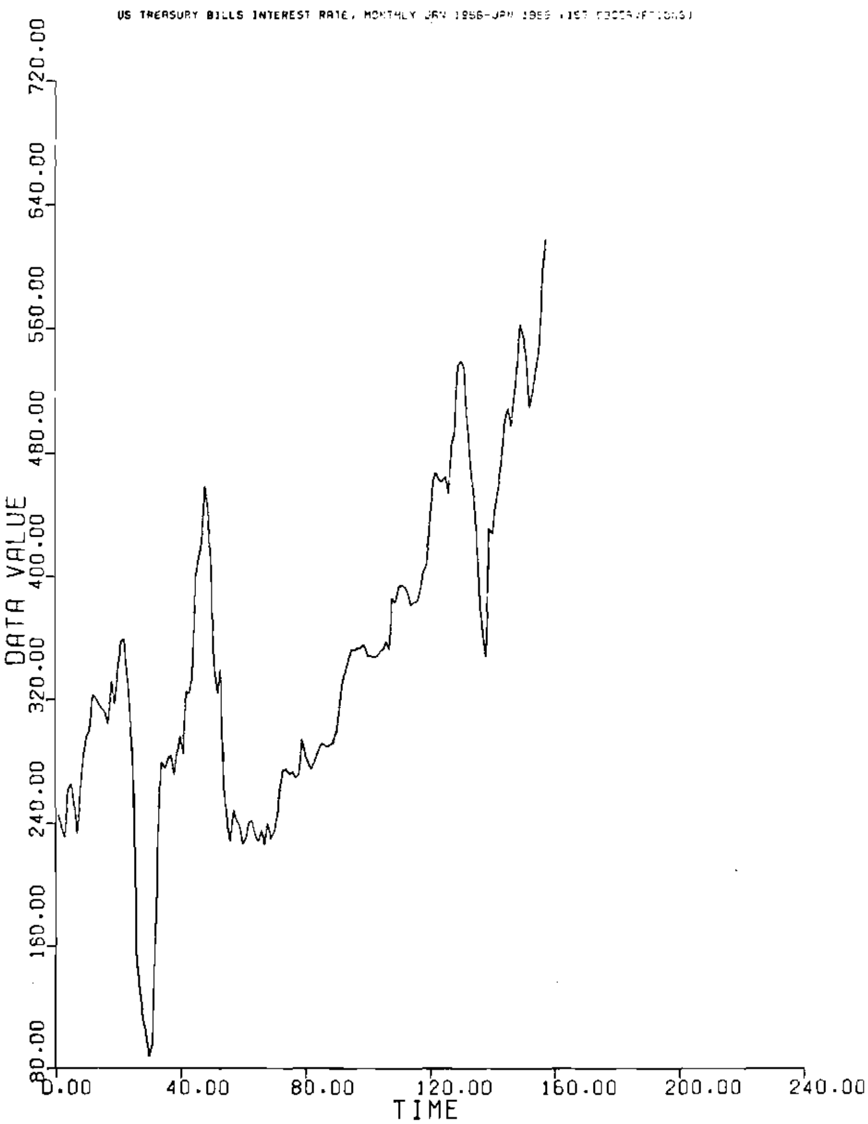


Figure 4.11. Series 11 U.S. Treasury Bills Interest Rate

Table 4.12 Alternative Forecasting Models for Series 11 - U.S. Treasury Bills Interest Rate, Monthly

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.2513$	ARIMA(2,1,0)	Slope=-0.1998	Second Order	$\hat{c}_1=1.0934$
$\hat{w}_2 = 0.1249$	p=2	Intercept=225.64	Intercept=259.46	$\hat{c}_2=0.9692$
$\hat{w}_3 = -0.0989$	d=1		Slope=4.26	$\hat{c}_3=0.9256$
$\hat{w}_4 = 0.1533$	q=0		S1 = 258.18	$\hat{c}_4=0.9120$
$\hat{w}_5 = 0.0310$	$\phi_1 = -0.0787$		S2 = 256.91	$\hat{c}_5=0.9053$
$\hat{w}_6 = -0.0386$	$\phi_2 = -0.48066$		$\alpha = 0.77$	$\hat{c}_6=0.8800$
$\hat{w}_7 = -0.1873$				$\hat{c}_7=0.8485$
$\hat{w}_8 = 0.0330$				$\hat{c}_8=0.9539$
$\hat{w}_9 = 0.0941$				$\hat{c}_9=1.0885$
$\hat{w}_{10} = 0.0749$				$\hat{c}_{10}=1.1318$
$\hat{w}_{11} = 0.2790$				$\hat{c}_{11}=1.1296$
$\hat{w}_{12} = 0.3240$				$\hat{c}_{12}=1.1622$
$k = 0.12$				$\alpha = 0.70$
80 iterations				$\beta = 0.70$
				$\gamma = 0.20$
				$\hat{b}_1(0)=268.6115$
				$\hat{b}_1(0)=-0.4505$
$MSE_I = 623.46$	1084.91	365.22	767.66	1453.45
$MSE_F = 323.45$	583.62	784.20	346.31	1311.65

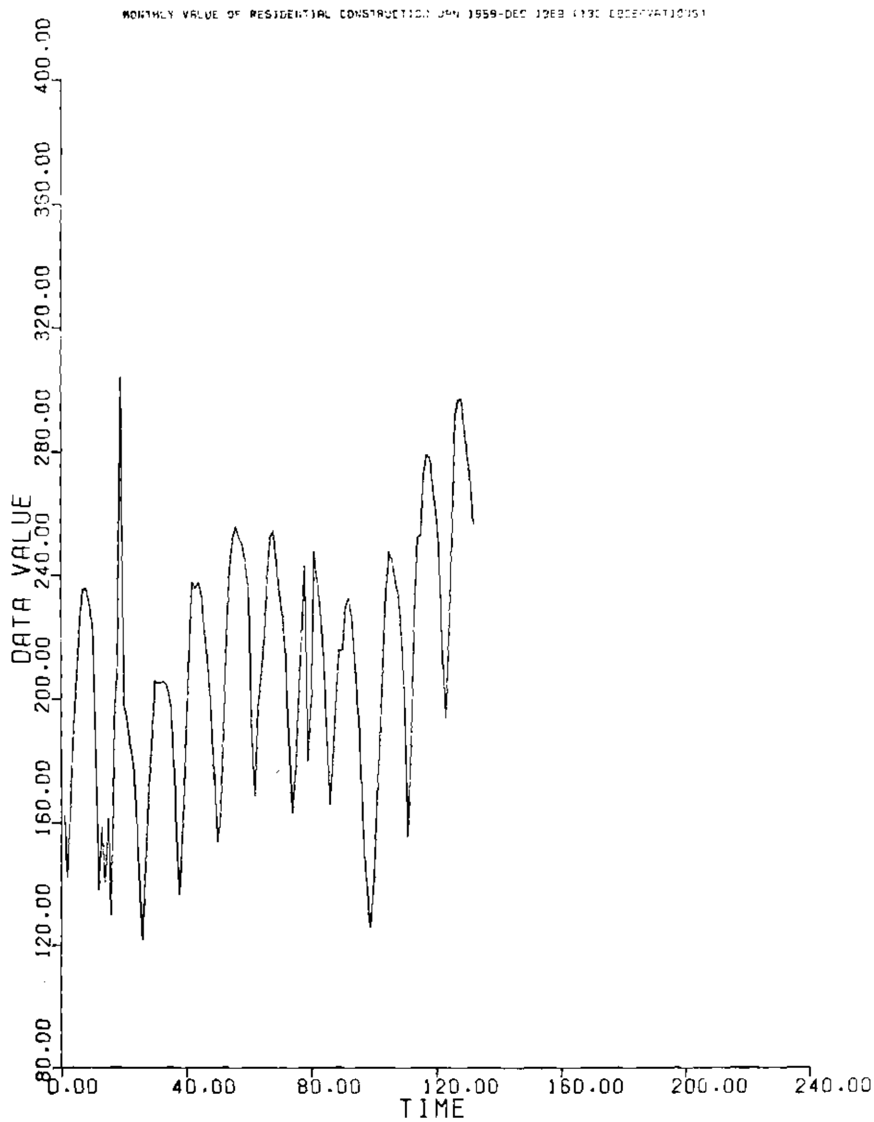


Figure 4.12. Series 12 Monthly Value of Residential Construction

Table 4.13 Alternative Forecasting Methods for Series 12 - Monthly Value of Residential Construction

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.2513$	Multiplicative	Slope=5.8613	Second Order	$\hat{c}_1$=0.8115
$\hat{w}_2 = 0.1249$	p=0	Intercept=1778.18	Intercept=221.29	$\hat{c}_2$=0.7143
$\hat{w}_3 = -0.0989$	d=1		Slope = 15.34	$\hat{c}_3$=0.8176
$\hat{w}_4 = 0.1533$	q=1		S1 = 211.89	$\hat{c}_4$=0.8907
$\hat{w}_5 = 0.0310$	P=0		S2 = 202.48	$\hat{c}_5$=1.0423
$\hat{w}_6 = -0.0386$	D=1		α = 0.62	$\hat{c}_6$=1.1420
$\hat{w}_7 = -0.01873$	Q=1			$\hat{c}_7$=1.2560
$\hat{w}_8 = 0.0330$	S=12			$\hat{c}_8$=1.1455
$\hat{w}_9 = 0.0941$	θ=0.78881			$\hat{c}_9$=1.1253
$\hat{w}_{10} = -0.0749$	Θ=0.5667			$\hat{c}_{10}$=1.0924
$\hat{w}_{11} = 0.2790$	Log taken			$\hat{c}_{11}$=1.0475
$\hat{w}_{12}$=0.3240				$\hat{c}_{12}$=0.9151
k = 0.09				α = 0.40
				β = 0.20
				γ = 0.20
				$\hat{b}_1(0)$=195.09
				$\hat{b}_2(0)$=0.3997
MSE_I = 352.89	504.63	128088.00	984.85	534.11
MSE_F = 688.34	503.81	*	573.33	526.02

*Values greater than 10^5 not considered

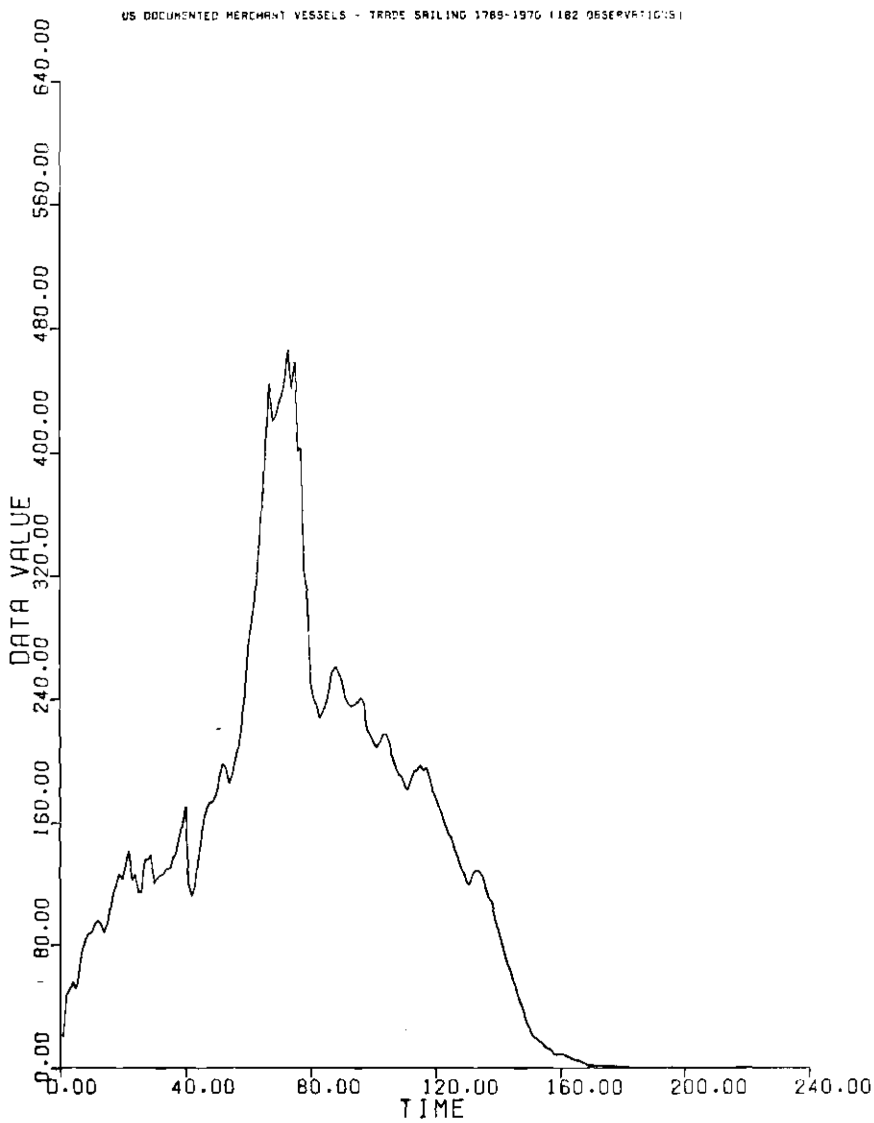


Figure 4.13. Series 13 U.S. Documented Merchant Vessels

Table 4.14 Alternative Forecasting Methods for Series 13 - U.S. Documented Merchant Vessels - Trade Sailing

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.9298$	ARIMA(1,2,0)	Slope=2.7857	Second Order	$\hat{c}_1 = 1.0$
$\hat{w}_2 = 0.0449$	p=1	Intercept=63.73	Intercept=64.33	$\alpha = 0.70$
k = 0.20	d=2		Slope = 6.60	$\beta = 0.50$
	ϕ_1=-0.63870		S1 = 60.46	$\gamma = 0.70$
	Log taken		S2 = 56.58	$\hat{b}_1(0) = 19.03$
			$\alpha = 0.63$	$\hat{b}_2(0) = 1.9394$
MSE_I = 267.117	304.336	5778.12	272.607	9.555
MSE_F = 16.128	9.569	*	11.817	11.536

*Values greater than 10^5 not considered

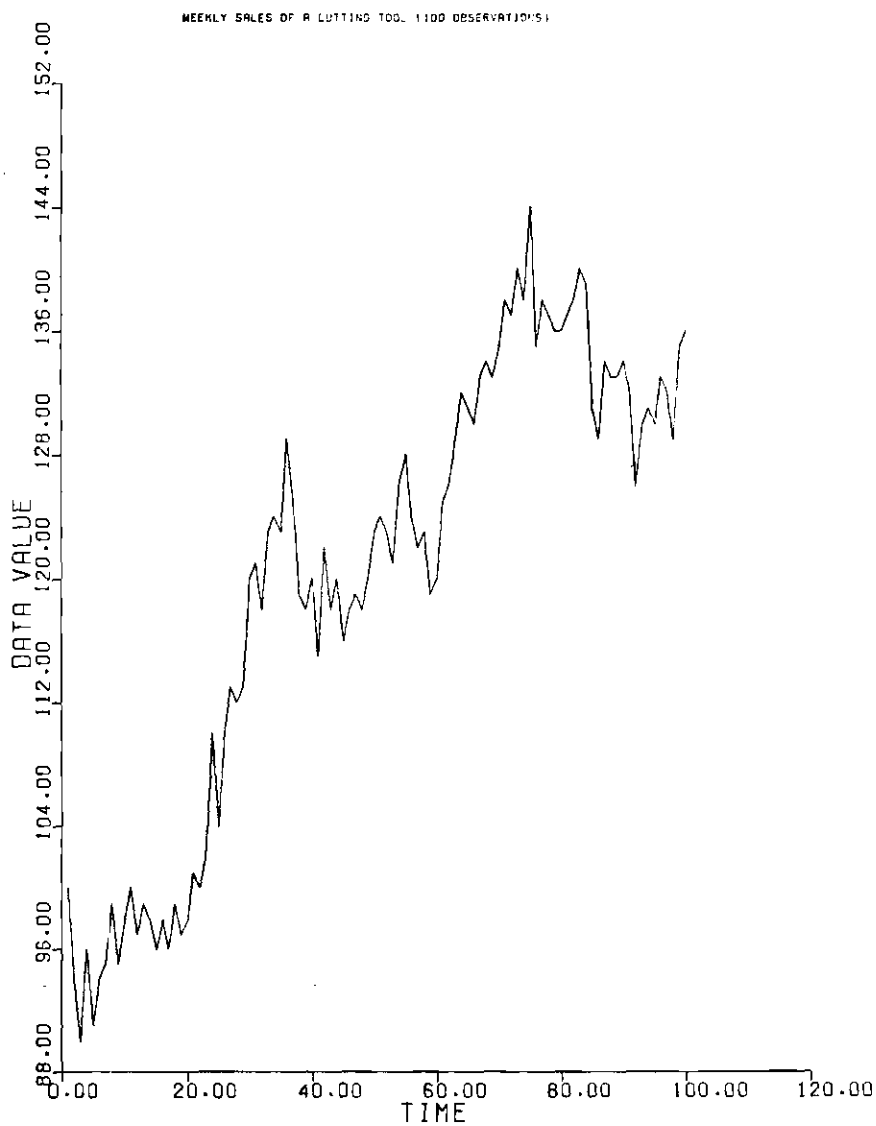


Figure 4.14. Series 14 Weekly Sales of a Cutting Tool

Table 4.15 Alternative Forecasting Methods for Series 14 - Weekly Sales of a Cutting Tool

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1$ = 0.2249	AR(1)	Slope=0.6116	First Order	$\hat{c}_1$=1.0162
$\hat{w}_2$ = 0.2764	p=-0.6233	Intercept=92.21	Intercept=94.17	$\hat{c}_2$=0.9906
$\hat{w}_3$ = 0.2996	Log used		S1 = 94.17	$\hat{c}_3$=0.9961
$\hat{w}_4$ = 0.2248			α = 0.30	$\hat{c}_4$=0.9970
k = 0.18				α = 0.70
				β = 0.20
				γ = 0.20
				$\hat{b}_1(0)$ = 93.8521
				$\hat{b}_2(0)$ = 0.5739
MSE_I = 10.198	19.281	23.877	16.413	4.488
MSE_F = 31.968	23.893	200.181	9.609	13.283

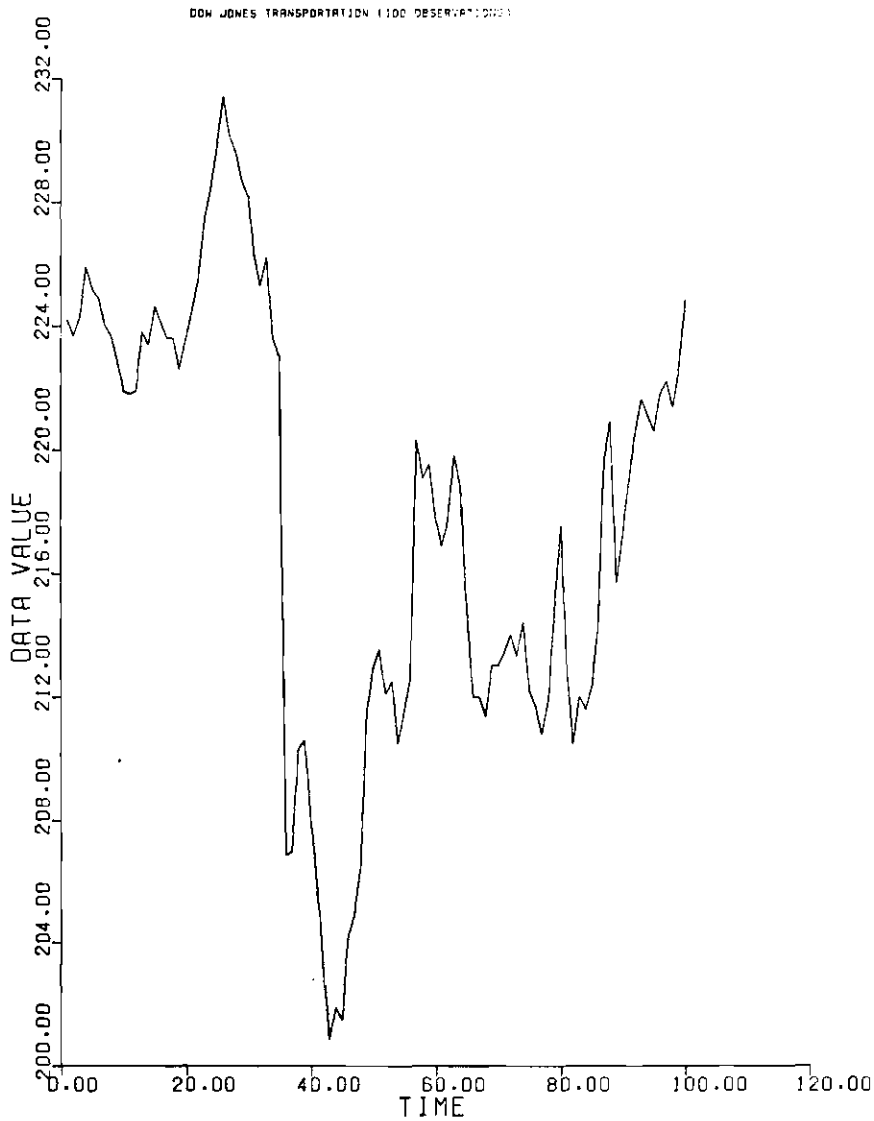


Figure 4.15. Series 15 Dow Jones Transportation

Table 4.16 Alternative Forecasting Methods for Series 15 Dow Jones

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1$ = 0.1405	ARIMA(1,1,0)	Slope=-0.1998	First Order	$\hat{c}_1$ = 1.00
$\hat{w}_2$ = 0.1980	p=1	Intercept=225.64	Intercept=224.70	α = 0.70
$\hat{w}_3$ = 0.2657	d=1		S1 = 224.70	β = 0.20
$\hat{w}_4$ = 0.3721	ϕ=0.17243		α = 0.30	γ = 0.70
k = 0.60	Log taken			$\hat{b}_1(0)$ = 224.28
				$\hat{b}_2(0)$ = -0.1622
MSE_I = 8.054	6.296	45.954	16.165	0.703
MSE_F =49.048	2.096	122.186	5.077	2.081

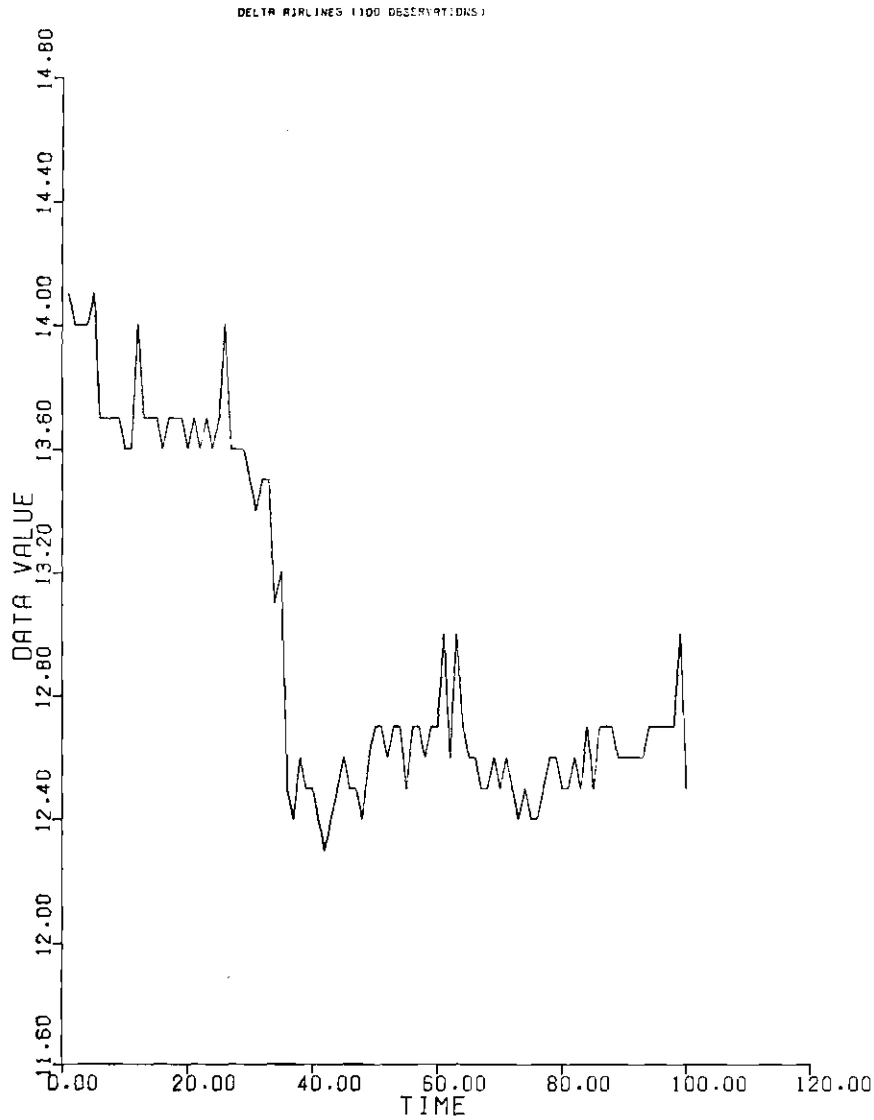


Figure 4.16. Series 16 Delta Airlines

Table 4.17 Alternative Forecasting Methods for Series 16 Delta Airlines

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.2403$	ARIMA(1,1,1)	Slope=-0.0233	First Order	$\hat{c}_1 = 1.0017$
$\hat{w}_2 = 0.2477$	p=1	Intercept=13.98	Intercept=13.98	$\hat{c}_2 = 1.0044$
$\hat{w}_3 = 0.2523$	d=1		S1 = 13.98	$\hat{c}_3 = 1.0049$
$\hat{w}_4 = 0.2544$	q=1		$\alpha = 0.30$	$\hat{c}_4 = 0.9999$
k = 0.20	$\phi = -0.0149$			$\hat{c}_5 = 1.0024$
	$\Theta = 0.5885$			$\hat{c}_6 = 0.9965$
	Log taken			$\hat{c}_7 = 0.9941$
				$\hat{c}_8 = 0.9995$
				$\hat{c}_9 = 1.0049$
				$\hat{c}_{10}=0.9950$
				$\hat{c}_{11}=1.0016$
				$\hat{c}_{12}=0.9952$
				$\alpha = 0.6$
				$\beta = 0.2$
				$\gamma = 0.2$
				$\hat{b}_1(0) = 13.9651$
				$b_2(0) = -0.0192$
$MSE_I = 0.032$	0.034	0.087	0.040	0.008
$MSE_F = 0.023$	0.014	0.559	0.013	0.022

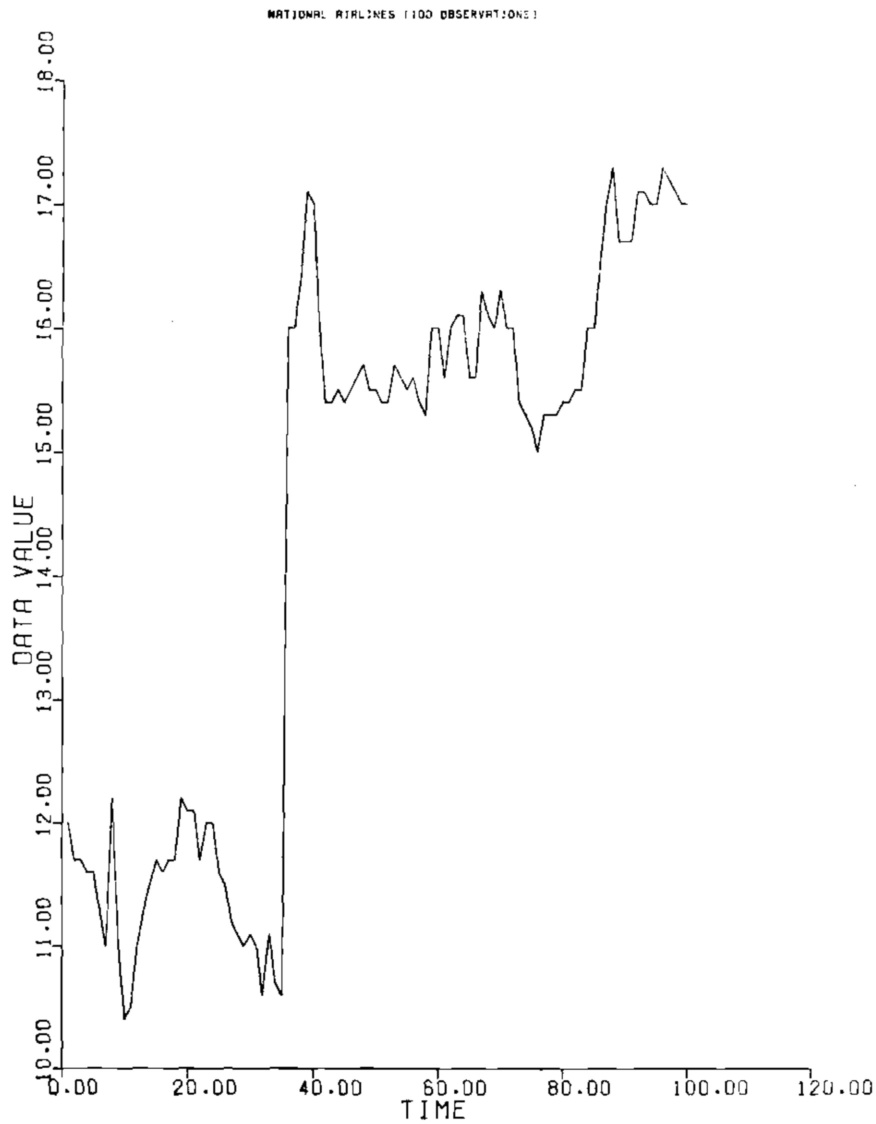


Figure 4.17. Series 17 National Airlines

Table 4.18 Alternative Forecasting Methods for Series 17 - National Airlines

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.5817$	IMA(1,1)	Slope=0.0849	First Order	$\hat{c}_1 = 1.0321$
$\hat{w}_2 = 0.4210$	p=0	Intercept=10.51	Intercept=11.65	$\hat{c}_2 = 1.0316$
k = 0.02	d=1		S1 = 11.65	$\hat{c}_3 = 1.0316$
	q=1		$\alpha = 0.30$	$\hat{c}_4 = 1.0204$
	$\Theta=0.0668$			$\hat{c}_5 = 1.0021$
				$\hat{c}_6 = 0.9882$
				$\hat{c}_7 = 0.9882$
				$\hat{c}_8 = 0.9931$
				$\hat{c}_9 = 0.9733$
				$\hat{c}_{10} = 0.9513$
				$\hat{c}_{11} = 0.9556$
				$\hat{c}_{12} = 1.0345$
				$\alpha = 0.70$
				$\beta = 0.20$
				$\gamma = 0.20$
				$\hat{b}_1(0) = 10.89$
				$\hat{b}_2(0) = 0.0737$
$MSE_I = 0.628$	0.544	1.664	0.981	0.068
$MSE_F = 0.073$	0.071	2.663	0.163	0.200

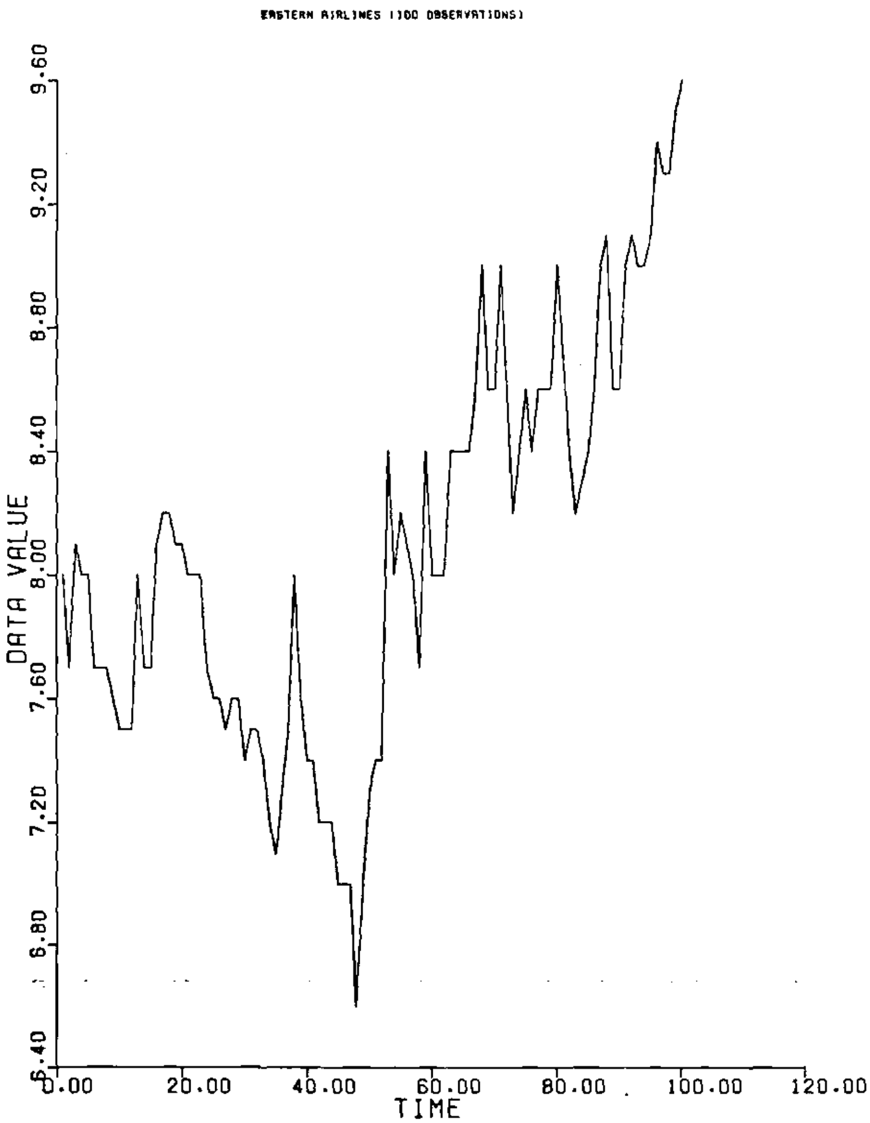


Figure 4.18. Series 18 Eastern Airlines

Table 4.19 Alternative Forecasting Methods for Series 18 - Eastern Airlines

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.2489$	AR(1)	Slope=0.0081	First Order	$\hat{c}_1 = 0.9958$
$\hat{w}_2 = 0.1700$	$\phi = 0.82291$	Intercept=7.51	Intercept=7.92	$\hat{c}_2 = 0.9995$
$\hat{w}_3 = 0.2415$	Mean=2.065		S1 = 7.92	$\hat{c}_3 = 1.0051$
$k = 0.20$			$\alpha = 0.30$	$\hat{c}_4 = 1.0074$
				$\hat{c}_5 = 1.0293$
				$\hat{c}_6 = 1.0037$
				$\hat{c}_7 = 1.0105$
				$\hat{c}_8 = 1.0146$
				$\hat{c}_9 = 0.9921$
				$\hat{c}_{10} = 0.9776$
				$\hat{c}_{11} = 0.9965$
				$\hat{c}_{12} = 0.9679$
				$\alpha = 0.70$
				$\beta = 0.20$
				$\gamma = 0.20$
				$\hat{b}_1(0) = 7.678$
				$\hat{b}_2(0) = 0.0119$
$MSE_I = 0.076$	0.068	0.222	0.087	0.025
$MSE_F = 0.124$	0.102	0.572	0.070	0.075

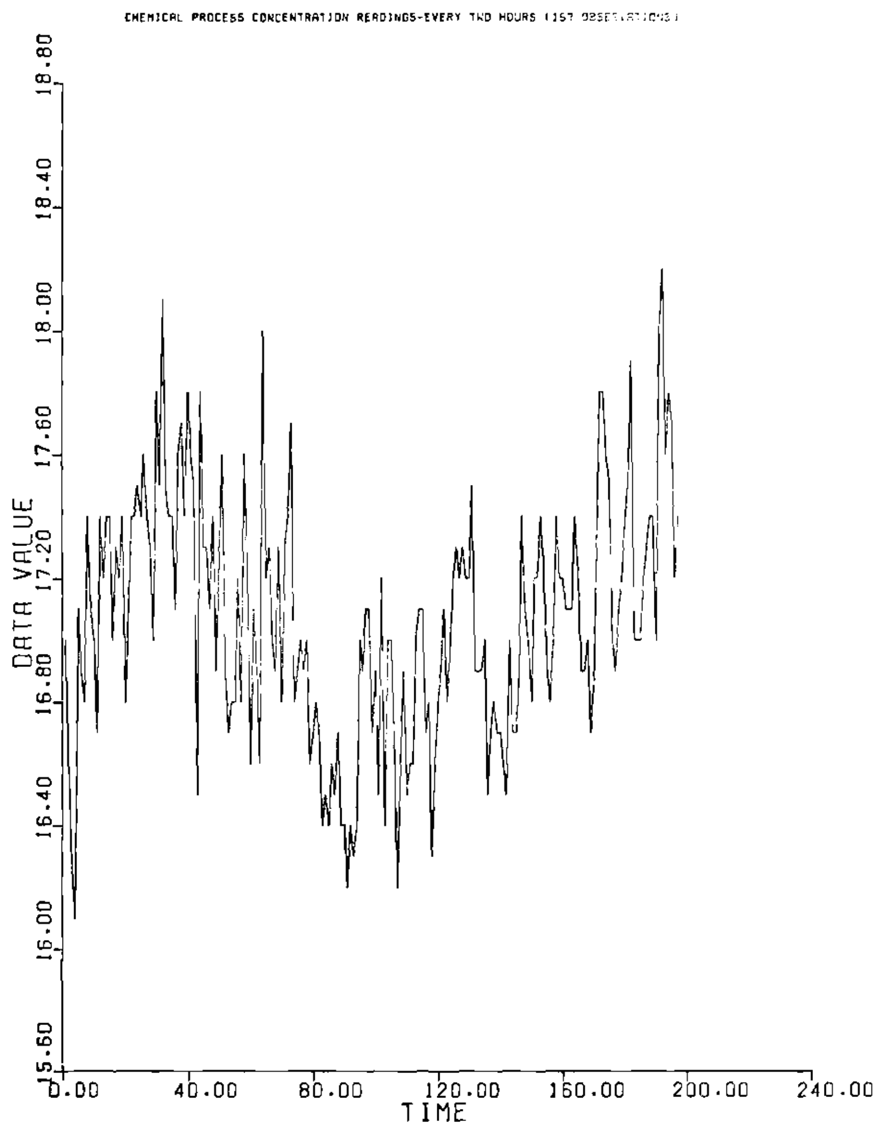


Figure 4.19. Series 19 Chemical Process Concentration Readings

Table 4.20 Alternative Forecasting Methods for Series 19 - Chemical Process Concentration Reading - Every Two Hours

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.5000$	ARIMA(1,1,0)	Slope= -0.0054	First Order	$\hat{c}_1 = 1.0$
$\hat{w}_2 = 0.5000$	p=1	Intercept= 17.33	Intercept=16.67	$\alpha = 0.20$
k = 0.0	d=1		S1 = 16.67	$\beta = 0.20$
	$\phi = -0.48972$		$\alpha = 0.28$	$\gamma = 0.20$
				$\hat{b}_1(0) = 17.0025$
				$\hat{b}_2(0) = -0.0010$
$MSE_I = 0.130$	0.117	0.574	0.109	0.095
$MSE_F = 0.098$	0.085	0.694	0.088	0.097

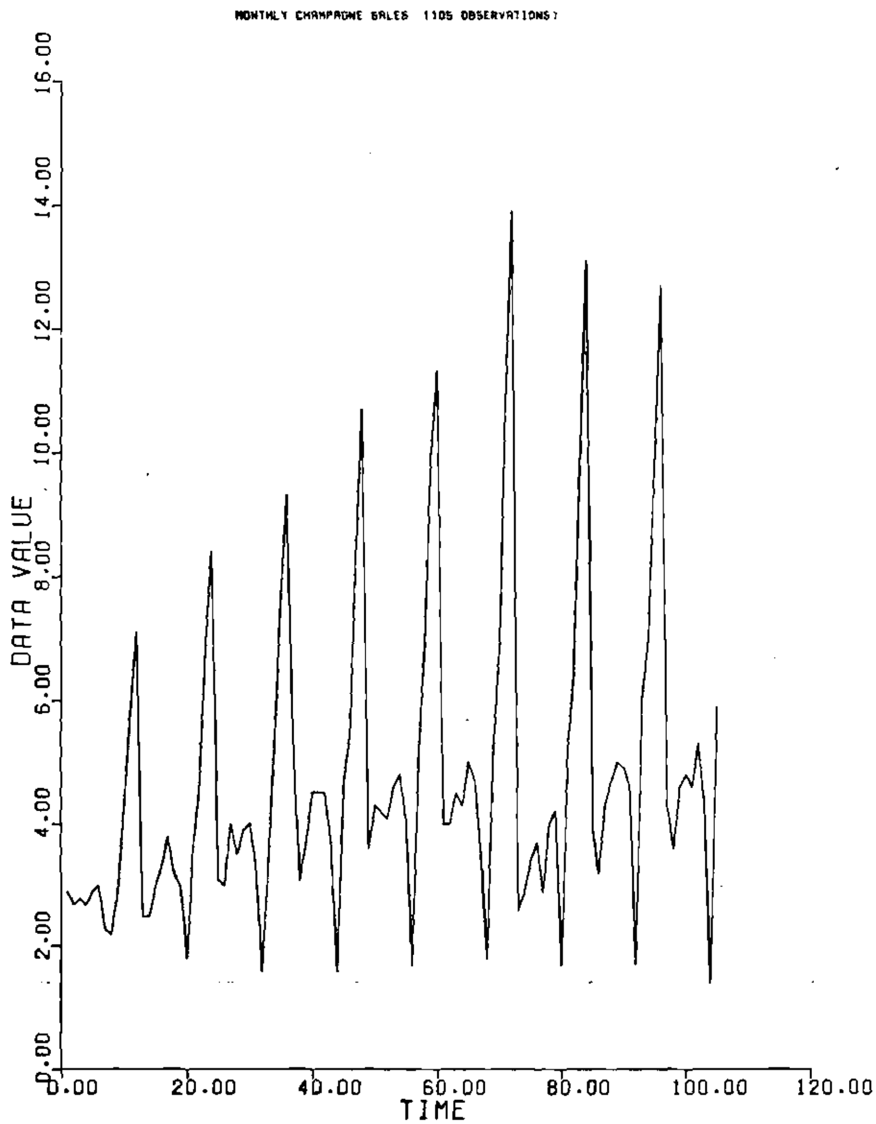


Figure 4.20. Series 20 Monthly Champagne Sales

Table 4.21 Alternative Forecasting Methods for Series 20 - Monthly Champagne Sales

Adaptive Filtering	Box & Jenkins	Linear Regression	Multiple Exponential Smoothing	Winters Method
$\hat{w}_1 = 0.0631$	Multiplicative	Slope = 0.0391	Third Order	$\hat{c}_1 = 0.9023$
$\hat{w}_2 = 0.0912$	p=0	Intercept = 3.07	Intercept = 3.02	$\hat{c}_2 = 1.2009$
$\hat{w}_3 = 0.0734$	d=0		Linear comp=-0.21	$\hat{c}_3 = 1.7628$
$\hat{w}_4 = -0.1377$	q=1		Quad. comp= 0.07	$\hat{c}_4 = 2.1636$
$\hat{w}_5 = 0.0852$	P=0		S1 = 5.49	$\hat{c}_5 = 0.7599$
$\hat{w}_6 = -0.0850$	D=1		S2 = 5.52	$\hat{c}_6 = 0.7033$
$\hat{w}_7 = 0.0935$	Q=2		S3 = 5.26	$\hat{c}_7 = 0.8114$
$\hat{w}_8 = 0.1153$	S=12		$\alpha = 0.13$	$\hat{c}_8 = 0.8288$
$\hat{w}_9 = 0.0760$	$\Theta = -0.32417$			$\hat{c}_9 = 0.8749$
$\hat{w}_{10} = -0.0943$	$\Theta_{12} = 0.18050$			$\hat{c}_{10} = 0.8791$
$\hat{w}_{11} = 0.1162$	$\Theta_{13} = 0.29904$			$\hat{c}_{11} = 0.7552$
$\hat{w}_{12} = 1.0186$	Log taken			$\hat{c}_{12} = 0.3578$
k = 0.08				$\alpha = 0.5$
				$\beta = 0.2$
				$\gamma = 0.2$
				$\hat{b}_1(0) = 3.449$
				$\hat{b}_2(0) = 0.0251$
$MSE_I = 0.449$	0.649	5.294	8.096	0.921
$MSE_F = 1.719$	0.525	11.523	10.092	2.271

CHAPTER V

ANALYSIS OF RESULTS

5.1 Comparison Between Individual Forecasting Methods in Fitting and Forecasting Phases

Each one of the twenty series of data was analyzed by all applicable forecasting methods. Results concerning the behavior of each forecasting method, measured using Mean Square Error (MSE) are shown in Tables 5.1 to 5.5. From these tables we can conclude that there is no "one best" forecasting method. The behavior of a method depends on the individual characteristics of the time series under study. For one method, the mean square error in the fitting phase may outperform all other methods, but there is no indication to believe that the same will happen in the forecasting phase.

When a forecasting method is used it is assumed that the pattern of the data will remain unchanged. For most of the time series in the study this was not the case. Actually, when using forecasting techniques to make a decision we do not know if the assumption of the same pattern will be appropriate. A method that reacts to changes in the pattern in the forecasting phase will be desired. In general, it is expected that if Box-Jenkins outperforms other methods fitting the data it will do the same when forecasting the future, (Table 5.7). Adaptive filtering is an example of a method that works very well in its fitting stage but does the contrary when forecasting for a lead time of one, refer to Series 3, 5, 7, 13 and 20 in Table 5.7

Table 5.1 Mean Square Error for the Adaptive Filtering Models for Different Lead Times

		Forecasting Phase Lead Times					
Series	Fitting Phase	1	2	3	6	9	12
1	101.6000	1350.1900	5055.4200	5893.2700	5051.1000	6335.2900	211.0100
2	38.3300	1091.4900	91.2000	94.1100	81.6500	91.4400	88.9000
3	22.1400	409.7800	33.0800	34.4800	34.6600	37.7900	37.8400
4	0.0883	0.1083	0.2294	0.3210	0.5042	0.5525	0.5198
5	4267.7800	9678.4100	10031.2600	11432.2900	10043.0200	13641.3500	5326.3200
6	151.0200	272.8000	265.3600	243.5600	227.6800	277.3100	230.7700
7	194.5700	397.4300	2536.8900	3489.9800	4296.9700	2236.9800	2375.1100
8	10.5900	39.2700	41.4900	40.6700	32.8200	48.2400	64.0700
9	0.0284	0.1273	0.0119	0.0189	0.0230	0.0280	0.0297
10	2550.8200	3753.7300	3158.2700	3404.6100	2855.6900	2438.4900	*
11	623.4600	323.4500	532.2600	431.4900	586.5400	624.9900	732.6400
12	352.9000	688.3400	*	*	*	*	*
13	267.1200	16.1300	41.3600	60.3300	81.9700	92.7900	133.8900
14	10.2000	31.9700	16.1500	13.7100	10.8700	17.7100	18.2300
15	8.0500	49.0500	2.7900	2.9800	2.9400	2.7400	3.0600
16	0.0316	0.0229	0.0163	0.0176	0.0158	0.0163	0.0224
17	0.6282	0.0736	0.1745	0.2517	0.5974	0.9498	1.2400
18	0.0764	0.1237	0.0986	0.0923	0.0681	0.0626	0.1087
19	0.1297	0.0976	0.1374	0.1558	0.1496	0.1629	0.1880
20	0.4487	1.7200	19.5000	22.5400	25.5600	30.6200	1.1000

*Greater than 10^5

Table 5.2 Mean Square Error for the Box-Jenkins Models for Different Lead Times

		Forecasting Phase Lead Times					
Series	Fitting Phase	1	2	3	6	9	12
1	85.1250	164.7393	304.2809	492.9825	660.5980	1283.0381	30112.9500
2	30.8471	77.6810	158.3683	241.0420	474.7089	786.6720	1129.7546
3	22.2860	30.2628	56.6990	87.9127	200.3601	339.4210	476.0185
4	0.0851	0.1116	0.2152	0.2921	0.4229	0.4528	0.5019
5	5421.4263	4305.2824	6958.3160	*	*	*	*
6	271.3759	90.6692	441.8446	352.2187	308.5061	158.1593	184.8608
7	244.7532	225.5748	761.1387	986.6007	921.5151	912.7267	836.8538
8	8.9335	23.0731	22.1117	21.2272	18.9529	17.1187	15.6083
9	0.0246	0.0119	0.0416	0.1030	0.4931	1.1878	1.9244
10	2495.0746	1214.1890	2523.6740	2283.3860	2227.2400	2173.7840	*
11	1084.9070	583.6171	1481.2244	1982.2860	3598.1780	4486.3500	5137.3556
12	504.6290	503.8135	638.8584	733.3814	1154.1700	3682.2870	10687.2400
13	304.3361	9.5688	39.0271	102.8323	513.3774	1026.0530	1820.5600
14	19.2812	23.8931	54.4483	77.2579	197.9327	337.0876	655.6657
15	6.2958	2.0962	4.4648	6.7561	17.8611	25.3329	34.2079
16	0.0336	0.0143	0.0133	0.0161	0.0210	0.0212	0.0298
17	0.5438	0.0707	0.1461	0.2198	0.4953	0.7929	1.0143
18	0.0685	0.1023	0.2587	0.3948	0.7264	0.9272	1.0430
19	0.1167	0.0848	0.1329	0.1521	0.1454	0.1495	0.1701
20	0.6490	0.5254	2.0534	2.0534	2.0534	2.0534	2.0534

*For lead times greater than 1, MSE greater than 10^5 were not considered

Table 5.3 Mean Square Error for the Linear Regression Models for Different Lead Times

Series	Fitting Phase	Forecasting Phase Lead Times: 1	2	3	6	9	12
1	718.0352	5559.5000	5663.1050	5829.6816	6413.8679	7084.5311	7646.9200
2	584.4846	70754.0000	*	*	*	*	*
3	193.8400	3112.8469	3109.9600	3107.0800	3098.4500	3089.8500	3081.2600
4	0.2309	0.9752	0.9642	0.9533	0.9215	0.8908	0.8611
5	8445.7400	*	*	*	*	*	*
6	176.1452	233.1280	233.4051	233.6846	234.5200	235.3844	236.2510
7	479.4600	1788.2480	1767.7360	1747.5090	1688.5300	1632.1100	1578.2440
8	181.0560	1637.8304	1578.5200	1520.7400	1356.5500	1206.0980	1069.3700
9	5.9650	3.4736	3.4637	3.4540	3.4250	3.3964	3.3683
10	2698.3170	2173.1690	2173.8500	2174.5500	2176.6700	2178.8500	2181.0900
11	365.2200	784.2000	799.6100	842.3500	965.2200	1342.1300	1466.0800
12	128088.0000	*	*	*	*	*	*
13	5778.1200	*	*	*	*	*	*
14	23.8770	200.1812	185.0796	170.7900	132.8010	102.1393	78.7923
15	45.9546	122.1862	118.3590	114.6200	103.9200	94.0050	84.8686
16	0.0865	0.5587	0.5253	0.4931	0.4036	0.3248	0.2566
17	1.6639	0.6626	2.3886	2.1304	1.4496	0.9100	0.5115
18	0.2219	0.5715	0.5829	0.5944	0.6299	0.6666	0.7047
19	0.5738	0.6942	0.5133	0.5075	0.4907	0.4744	0.4586
20	5.2928	11.5235	11.3900	11.2782	10.9347	10.6207	10.3361

*Values greater than 10^5 not considered

Table 5.4 Mean Square Error for the Exponential Smoothing Models for Different Lead Times

Series	Fitting Phase	Forecasting Phase Lead Times 1	2	3	6	9	12
1	559.7980	2799.7300	9329.3800	18816.0000	46782.3000	82700.0600	75961.0000
2	45.2400	112.6680	243.8850	395.3760	893.6370	1718.7900	2963.0000
3	30.3934	36.1491	84.2380	148.1490	379.7270	755.5100	1149.8300
4	0.0833	0.1121	0.2182	0.3027	0.4549	0.3113	0.2946
5	5631.8800	12256.9000	19444.4000	25195.1000	43221.4000	83707.7700	116446.0000
6	182.5220	199.2700	211.9510	201.4360	194.8830	199.6850	219.5400
7	553.9270	481.6310	2354.1300	2435.5700	2860.5400	2484.7700	3332.3400
8	56.1539	45.4540	71.9039	100.7070	216.8200	368.5890	516.5970
9	0.5145	0.2832	0.4719	0.6975	1.4986	2.3166	3.0767
10	3494.6800	2863.9800	3957.7000	3816.6800	4166.2400	3535.6000	8031.4000
11	76766.1000	34630.5000	1075.7500	2418.1400	3328.0000	4238.4000	4904.4900
12	984.8500	573.3340	1980.2000	2267.9100	2591.9900	2102.8500	3183.3200
13	272.6070	11.8170	41.9656	102.2200	391.5300	644.2170	938.8500
14	16.4136	9.6093	11.7260	12.0729	14.8183	24.8716	37.2510
15	16.1652	5.0767	7.9711	11.4349	23.0606	28.1205	28.4510
16	0.0401	0.0126	0.0118	0.0132	0.0161	0.0188	0.0270
17	0.9812	0.1629	0.2560	0.3589	0.7256	1.0848	1.3127
18	0.0865	0.0697	0.0975	0.1113	0.1493	0.1959	0.2546
19	0.0883	0.0883	0.1091	0.1184	0.1287	0.1409	0.1435
20	8.0963	10.0923	13.1033	15.1400	23.3200	28.7058	27.6347

Table 5.5 Mean Square Error for Winters' Models for Different Lead Times

		Forecasting Phase Lead Times					
Series	Fitting Phase	1	2	3	6	9	12
1	332.4800	398.0680	904.6960	1550.1500	2968.0400	2926.7300	2005.5700
2	2107.8730	102.1660	205.9870	375.5040	680.4270	1189.1900	15824.6000
3	24.4650	33.6107	62.7341	121.3620	203.2980	580.1110	672.1460
4	0.1307	0.1386	0.2526	0.2812	0.6229	0.8053	1.0323
5	4771.9800	4735.8300	5551.5200	6962.7900	11290.2000	17962.1000	20169.5000
6	78.8366	233.3560	265.5040	227.6550	217.0270	227.6450	231.1430
7	5148.2300	15238.7000	39682.4000	170730.0000	67876.6000	18856.6000	*
8	7.3833	21.8546	42.3429	52.9748	101.4790	186.4580	328.0340
9	0.0126	0.0124	0.1932	0.2797	0.6825	1.2925	2.0865
10	3376.1400	3348.0100	3452.7600	3706.9300	4159.0300	4032.5300	7272.8000
11	1453.4500	1311.6500	4340.2200	8698.3900	16014.9000	16157.1000	15824.6000
12	534.1140	526.0210	878.6150	1119.7900	1799.7100	2020.1600	2534.3100
13	9.5549	11.5358	136.3450	193.5370	442.4430	638.9680	673.8400
14	4.4875	13.2832	21.4580	26.5388	35.8108	79.8930	130.1090
15	0.7030	2.0809	5.2283	7.1021	14.5753	25.8647	44.0541
16	0.0075	0.0223	0.0197	0.0292	0.0399	0.0743	0.1011
17	0.0678	0.2006	0.4940	0.8173	2.3808	2.4665	3.1514
18	0.0252	0.0747	0.1218	0.1414	0.2358	0.3801	0.5576
19	0.0954	0.0974	0.1149	0.1243	0.1254	0.1602	0.2145
20	0.9206	2.2710	0.7808	1.0602	2.5877	1.7487	1.6809

*Values greater than 10^5 not considered

Table 5.6 Comparative Results Between Methods - Entries in the Table are Mean Square Error

Series	Forecasting Method	Fitting Phase	Forecasting Phase Lead Times (τ) τ=1	τ=2	τ=3	τ=6	τ=9	τ=12
1	Adaptive	101.6000	1350.1900	5055.4200	5893.2700	5051.1040	6335.2900	211.0100
	Box & Jenkins	85.1250	164.7393	304.2809	492.9825	660.5988	1283.0381	30112.9500
	Regression	718.0352	5559.5000	5663.1050	5829.6316	6413.8679	7034.5311	7646.9200
	Smoothing	559.7980	2799.7300	9329.3800	18816.0000	46782.3000	82700.0600	75961.000
	Winters	332.4800	398.0680	904.6960	1550.1500	2968.0400	2926.7300	2005.5700
2	Adaptive	38.3267	1091.4900	91.1991	94.1066	81.6494	91.4439	88.9008
	Box & Jenkins	30.8471	77.6810	158.3683	241.0420	474.7089	786.6720	1129.7546
	Regression	584.4866	70754.6000	*	*	*	*	*
	Smoothing	45.2400	112.6680	243.8850	395.3760	893.6370	1718.7900	2936.1500
	Winters	107.8730	102.1660	205.9870	375.5040	680.4270	1189.1900	15824.6000
3	Adaptive	22.1455	409.7800	33.0825	34.4832	34.6640	37.7867	37.8411
	Box & Jenkins	22.2860	30.2628	56.6990	87.9127	200.3601	339.4210	476,0185
	Regression	193.8400	3112.8469	3109.9600	3107.0800	3098,4500	3089.8500	3081.2600
	Smoothing	30.3934	36.1491	84,2380	148,1490	397.7270	755,5100	1149.8300
	Winters	24.4650	33.6107	62.7341	121,3620	283.2980	580.1110	672.1460
4	Adaptive	0.0883	0.1083	0.2294	0.3210	0,5042	0,5525	0.5198
	Box & Jenkins	0.0851	0.1116	0.2152	0.2921	0,4229	0.4528	0.5019
	Regression	0.2309	0.9752	0.9642	0.9533	0.9215	0.8908	0.8611
	Smoothing	0.0833	0.1121	0,2182	0,3027	0,4549	0.3113	0.2946
	Winters	0.1307	0.1386	0,2526	0,2812	0.6229	0.8053	1.0323

*Greater than 10^5

Continuation of Table 5.6

Series	Forecasting Method	Fitting Phase	Forecasting Phase Lead Times (τ) τ=1	τ=2	τ=3	τ=6	τ=9	τ=12
5	Adaptive	4267.7800	9678.4100	10031.2600	11432.2900	10043,0230	13641.3500	5326.3200
	Box & Jenkins	5421.4300	4305.2800	6958.3200	*	*	*	*
	Regression	8445.7400	*	*	*	*	*	*
	Smoothing	5631.8800	12256.9000	19444.4000	25195.1000	43221.4000	83707,7700	11644.6000
	Winters	4771.9800	4735.8300	5551.5200	6962.7900	11290.2000	17962.1000	20169.5000
6	Adaptive	151.0165	272.8010	265.3620	243.5650	227,6831	277,3080	230.7700
	Box & Jenkins	271.3759	90,6692	441.8446	352,2187	308,5061	158.1593	184,8608
	Regression	176.1452	233,1280	233.4051	233,6846	234.5200	234.3544	236,2510
	Smoothing	182,5220	199.2700	211,9510	201,4360	194.8830	199,6850	219.5340
	Winters	78.8366	233.3560	265,5040	227,6550	217,0290	227,6450	231.1430
7	Adaptive	194,5730	397,4300	2536,8900	3489.9800	4296.9700	2236.9800	2375.1100
	Box & Jenkins	244.7532	255.5748	761.1387	986.6007	921.5151	912.7267	836.8538
	Regression	479.4600	1788.2480	1767,7360	1747,5090	1688.5300	1632,1100	1578.2440
	Smoothing	553.9270	481,6310	2354,1300	2435.5700	2860.5400	2484,7700	3332.3400
	Winters	5148.2300	15238,7000	39682,4000	170730,0000	67876,6000	18856,6000	*
8	Adaptive	10.5939	39,2661	41.4862	40,6700	32,8236	48.2452	64.0726
	Box & Jenkins	8.9335	23.0731	22,1117	21.2272	18,9529	17.1187	15.6083
	Regression	181.0560	1637,8304	1578.5200	1520.7400	1356,5500	1206,0980	1069.3700
	Smoothing	56.1539	45.4541	71.9039	100.7070	216.8200	368.5890	516,5170
	Winters	7.38332	21.8546	42,3429	52,9748	101.4790	186.4580	328.0340

*Greater than 10^5

Continuation of Table 5.6

Series	Forecasting Method	Fitting Phase	Forecasting Phase Lead Times (τ) τ=1	τ=2	τ=3	τ=6	τ=9	τ=12
9	Adaptive	0.0284	0.1273	0.0119	0,0189	0.0230	0.0280	0.0297
	Box & Jenkins	0,0246	0.0119	0.0416	0,1030	0,4931	1,1878	1.9244
	Regression	5.9650	3.4736	3.4637	3.4540	3.4250	3.3964	3.3683
	Smoothing	0.5145	0.2832	0.4719	0.6976	1.4986	2.3167	3.0767
	Winters	0.0126	0.0124	0,1932	0,2797	0.6825	1.2925	2.0865
10	Adaptive	2550.8200	3753.7300	3158.2680	3404.6100	2855.6900	2438.4900	*
	Box & Jenkins	2495.0746	1214.1890	2523.6740	2283.3860	2227.2400	2173.7840	2106.3700
	Regression	2698.3170	2173.1690	2173,8500	2174.5500	2176.6700	2178.8500	2181.0900
	Smoothing	3494.6800	2863.9800	3957.7000	3816.6800	4166.2400	3535.6000	8031.4000
	Winters	3376.1400	3348.0100	3452.7600	3706.9300	4159.0300	4032.5300	7277.8000
11	Adaptive	623.4600	323.4500	532.2600	431,4900	586,5400	624.9900	732.6400
	Box & Jenkins	1084.9070	583.6171	1481.2244	1982.2860	3598.1780	4486.3500	5137,3556
	Regression	365.2200	784.2000	799.6100	842,3500	965.2200	1342.1300	1466.0800
	Smoothing	767.6610	346.3050	1075.7500	2418,1400	3328,0000	4238,4000	4904.4900
	Winters	1453.4500	1311.6500	4340.2200	8698,3900	16014,9000	16157.1000	15824.6000
12	Adaptive	352.9000	688,3400	*	*	*	*	*
	Box & Jenkins	504.6290	503.8135	638.8584	733,3814	1154,1706	3682,2870	10687.2400
	Regression	128088.0000	*	*	*	*	*	*
	Smoothing	573.3340	573.3340	1980.2000	2267,9100	2591.9900	2102.8500	3183.3200
	Winters	534.1140	526.0210	878,6150	1119.7900	1799,7100	2020.1600	2534.3100

*Greater than 10^5

Continuation of Table 5.6

Series	Forecasting Method	Fitting Phase	Forecasting Phase Lead Times (τ) τ=1	τ=2	τ=3	τ=6	τ=9	τ=12
13	Adaptive	267.1170	16.1270	41.3576	60.3300	81,9700	92,7900	133.8900
	Box & Jenkins	304.3361	9.5688	39.0271	102.8323	513,3770	1026,0530	1820.5600
	Regression	5778.1200	*	*	*	*	*	*
	Smoothing	272.6070	11,8174	41.9656	102,2200	391,5300	644,2170	938.8500
	Winters	9.5548	11.5358	136.3450	193,5370	442,4430	638.9680	673,8400
14	Adaptive	10.1983	31.9675	16.1544	13.7138	10.8661	14,7082	18.2300
	Box & Jenkins	19.2812	23.8931	54,4483	77,2579	197,9327	337,0876	655,6657
	Regression	23.8770	200.1812	185.0790	170,7900	132.8010	102,1390	78.7923
	Smoothing	16.4136	9.6093	11,7260	12,0729	14.8183	24.8716	37,2510
	Winters	4.4875	13.2832	21.4580	26.5388	35.8108	79.8930	130.1090
15	Adaptive	8.0545	49.0489	2.7901	2,9780	2,8398	2.7441	3.0656
	Box & Jenkins	6.2958	2.0962	4.4648	6,7561	17.8611	25,3329	34,2079
	Regression	45.9546	122.1862	118.3590	114.6200	103.9200	94,0050	84,8686
	Smoothing	16.1652	5.0767	7.9711	11,4349	23.0606	28.1205	28,4510
	Winters	.7030	2.0809	5.2280	7.1020	14,5753	25.8647	44.0541
16	Adaptive	0.0316	0.02290	0.0163	0.0176	0,0158	0.0163	0.0224
	Box & Jenkins	0.0336	0.0143	0,0133	0.0161	0.0210	0,0212	0.0298
	Regression	0.0865	0.5587	0.5253	0.4931	0.4036	0.3248	0.2566
	Smoothing	.0400	.0125	.0118	,0132	.0161	.0188	,0278
	Winters	.0075	.0223	.0197	.0292	.0399	.0743	.1011

*Greater than 10^5

Continuation of Table 5.6

Series	Forecasting Method	Fitting Phase	Forecasting Phase Lead Times (τ) τ=1	τ=2	τ=3	τ=6	τ=9	τ=12
17	Adaptive	0.6282	0.0735	0.1745	0.2517	0.5974	0.9498	1.2423
	Box & Jenkins	0.5438	0.0707	0.1461	0.2198	0.4953	0.7929	1,0143
	Regression	1.6639	2.6626	2.3886	2.1304	1.4496	0.9100	0.5115
	Smoothing	0.9812	0.1629	0.2563	0.3589	0.7257	1.0848	1.3128
	Winters	0.0678	0.2006	0.49405	0.8173	2.3803	2.4664	3.1514
18	Adaptive	0.0764	0.1237	0.0986	0.0923	0.0681	0.0626	0.1087
	Box & Jenkins	0.0685	0.1023	0.2587	0.3948	0.7264	0,9272	1,0430
	Regression	0.2219	0.5715	0.5829	0.5944	0.6299	0.6666	0.7047
	Smoothing	0.0865	0.0697	0.0975	0.1113	0.1493	0.1959	0,2546
	Winters	0.0252	0.0746	0.1218	0.1414	0.2358	0.3801	0.5576
19	Adaptive	0.1297	0.0976	0.1374	0.1558	0.1496	0.1629	0.1880
	Box & Jenkins	0.1167	0.0848	0.1329	0.1521	0.1454	0.1495	0.1701
	Regression	0.5738	0.6942	0.5133	0,5075	0.4907	0,4744	0.4586
	Smoothing	0.1086	0.0883	0.1091	0,1184	0.1287	0,1409	0,1435
	Winters	0.0954	0.0974	0.1148	0.1243	0.1254	0.1602	0.2145
20	Adaptive	0.4487	1.7192	19.5000	22.5450	25.5647	30.6200	1.0999
	Box & Jenkins	0.6490	0.5254	2.0534	2.0534	2.0534	2.0534	2.0534
	Regression	5.2928	11.5235	11.3900	11.2780	10,9350	10.6210	10,3360
	Smoothing	8.0963	10.0923	13.1033	15.1400	23,3200	28.7058	27.6347
	Winters	0.9206	2.2710	0.7808	1.0602	2.5878	1.7487	1.6809

Table 5.7 Best Forecasting Method in Different Lead Times (Selection Based on MSE)

Series	Fitting Phase	Forecasting Lead Times: 1	2	3	6	9	12
1	B	B	B	B	B	B	A
2	B	B	A	A	A	A	A
3	A	B	A	A	A	A	A
4	S	A	B	W	B	S	S
5	A	B	W	W	A	A	A
6	W	B	S	S	S	B	B
7	A	B	B	B	B	B	B
8	B	W	B	B	B	B	B
9	W	B	A	A	A	A	A
10	B	B	R	R	R	R	R
11	R	B	R	R	R	R	R
12	B	B	B	B	B	B	W
13	A	B	B	A	A	A	A
14	W	S	S	S	A	A	A
15	W	W	A	A	A	A	A
16	W	S	S	S	A	A	A
17	W	B	B	B	B	B	R
18	W	S	S	S	A	A	A
19	W	B	S	S	W	S	S
20	A	B	W	W	B	W	A

SUMMARY Number of times each method out-performs the others for each lead time

Method	Fitting Phase	Forecasting Lead Times: 1	2	3	6	9	12	TOTALS
A	5	1	4	5	9	9	11	44
B	5	14	7	5	7	5	3	46
W	8	2	2	3	1	2	1	19
R	1	0	2	2	2	2	3	12
S	1	3	5	5	1	2	2	19
TOTALS	20	20	20	20	20	20	20	140

A = Adaptive Filtering
B = Box Jenkins
W = Winters Method
S = Smoothing
R = Regression

It is expected that the Mean Square Error will increase when the lead time increases because the information used in the forecasting phase is not as new as would be if a lead time of one were used. The Box-Jenkins approach is, in general, superior to the other forecasting techniques for short lead times, but part of this advantage is lost when forecasting for lead times greater than one (Table 5.2). Comparative results of the performance of the forecasting methods for each series are shown in Table 5.6. Table 5.7 summarizes the results from Table 5.6, showing by series what forecasting method outperforms the others. Detailed analysis of this table shows that Adaptive Filtering and Box-Jenkins are the forecasting methods that perform best for the time series studied. However, a different pattern can be noticed in the results of these two methods. Adaptive Filtering does not perform well for a lead time of one, while Box-Jenkins does. The number of times that Box-Jenkins outperforms the other methods decreases as lead time increases, contrary to Adaptive Filtering.

5.2 Combination of Forecasts

Often, when analyzing a time series, the analyst must choose between two forecasting methods. If the choice between these two methods is not completely evident, the selection of one of them will represent a loss of information contained in the rejected method. The results of combining the two methods, will now be investigated. In this analysis, only those series for which the selection between methods was not obvious were considered for the combination. For example, for Series 1, Box-Jenkins is the best method for both fitting and forecasting the data, and no other methods are reasonable contenders.

Table 5.8 presents the results of combinations of only two methods. If two methods are combined, the percent of improvement of the combined forecast over the best individual forecast is defined as

$$\% \text{ improvement} = \frac{\min\,(\sigma_1^2,\ \sigma_2^2) - V(e_{T+\tau}(T))}{\min\,(\sigma_1^2,\ \sigma_2^2)}$$

A plot of percent of improvement as a function of the correlation coefficient for the data in Table 5.8 appears in Figure 5.1.

Figure 5.1 shows that for the combination of two forecasting methods, when the correlation coefficient (ρ) between the forecast errors is near one, the percent of improvement is generally small, while when ρ is close to zero, improvements over the best single forecasting method reach approximately 50%.

From Table 5.8 we can conclude that a combination of two methods having a completely different behavior will do better than a combination of two that behave approximately the same. As an example for Series 4, the modeling phase for Adaptive Filtering, Box-Jenkins and Smoothing are close in variance of forecast error (Adaptive Filtering: 0.0883, Box-Jenkins: 0.0851 and Smoothing: 0.0834). A combination of methods, Box-Jenkins and Smoothing, gives almost no improvement in variance (.19% from Table 5.8). The best combination of two methods occur for the combination of Winters' and Smoothing which perform very differently, Smoothing being the best individual forecasting method for the modeling phase with $\sigma_e^2 = 0.083$ and Winters' with $\sigma_e^2 = .1307$.

For the series used in the combination, in general Adaptive

Table 5.8 Relationship Between Correlation Coefficient (ρ), Variance of Forecast Error (σ_e^2), and Percent of Improvement Over Minimum Variance (%) for the Combination of Two Forecasting Methods

Series		A-B	A-W	A-R	A-S	B-W	B-R	B-S	W-R	W-S	R-S	Best Combination
4	ρ	0.9848	0.5650	0.2546	0.9691	0.5675	0.3545	0.9947	0.1550	0.5964*	0.3728	
	σ_e^2	0.0846	0.0816	0.0789	0.0843	0.0785	0.0798	0.0841	0.0947	0.0782	0.0795	W-S
	%	0.49	9.84	12.77	0.08	8.28	6.83	0.19	26.39	7.34	5.81	
9	ρ	0.8969	0.0699	-0.1548	-0.0955	0.0226*	-0.1791	0.2163	0.1578	-0.0396	0.0943	
	σ_e^2	0.0246	0.0174	0.0280	0.0256	0.0151	0.0235	0.0254	3.4564	0.0341	0.5412	B-W
	%	0.29	40.95	4.98	9.94	39.47	5.83	0.00	42.05	8.74	5.23	
13	ρ	X	0.0157*	X	0.9547	X	X	X	X	0.0183	X	
	σ_e^2		146.747		280.43					148.05		A-W
	%		48.21		0.36					47.39		
14	ρ	0.7047	0.5217	0.1441*	0.6561	0.5156	0.2023	0.4625	0.1123	0.3730	0.5957	
	σ_e^2	10.5461	10.7696	8.4376	9.2915	18.9623	12.9155	10.4750	17.2407	11.8363	11.9333	A-R
	%	0.89	0.24	21.85	11.91	2.34	33.92	13.45	27.79	2.20	1.39	
15	ρ	0.8538	-0.0396	-0.0165	0.3108	-0.1818*	0.1592	0.6220	-0.0425	-0.1821	0.4857	
	σ_e^2	6.4022	3.8643	7.0753	6.9629	2.8223	6.0584	6.6848	38.9834	6.9957	16.2359	B-W
	%	0.14	49.28	17.04	14.68	55.86	4.87	0.04	7.94	57.38	1.65	
16	ρ	0.7974	0.8053	0.1507	0.6517	0.7782	0.4421	0.9505	0.1083*	0.6476	0.5460	
	σ_e^2	0.0289	0.0287	0.0276	0.0284	0.0261	0.0302	0.0310	0.0231	0.0262	0.0360	W-R
	%	8.04	0.50	17.56	13.23	7.19	2.72	0.62	17.14	9.21	0.91	
17	ρ	0.8953	0.8053	0.2717	0.7917	0.8023	0.2715	0.6767	0.0997*	0.5516	0.6532	
	σ_e^2	0.5487	0.5719	0.5819	0.6944	0.5185	0.5014	0.5778	0.4806	0.5931	0.9560	W-R
	%	0.25	6.40	12.20	0.89	6.39	9.13	1.90	20.31	9.49	1.48	
18	ρ	0.7411	0.7919	0.1213	X	0.7290	0.5541	X	0.1332*	X	X	
	σ_e^2	0.0635	0.0625	0.0654		0.0573	0.0695		0.0553			W-R
	%	7.08	2.57	19.02		9.87	0.0		14.49			
19	ρ	0.9999	0.0867	0.5969	0.9132	0.0872	0.5968	0.9135	-0.1971	0.0247*	0.7353	
	σ_e^2	0.1338	0.0687	0.1146	0.1118	0.0687	0.1146	0.1118	0.1330	0.0672	0.1035	W-S
	%	0.00	42.20	15.24	0.29	42.19	15.23	0.28	15.49	40.11	7.68	
20	ρ	0.7219	0.6979*	0.3062	0.2664	0.7460	0.5712	0.5495	0.4725	0.5311	X	
	σ_e^2	0.4219	0.3915	0.4771	0.4770	0.4171	0.4710	0.4638	0.4095	0.4072		A-W
	%	10.16	12.44	0.06	0.09	6.70	10.09	11.47	4.44	10.73		

*Best combination of two methods

An X means combination not performed.

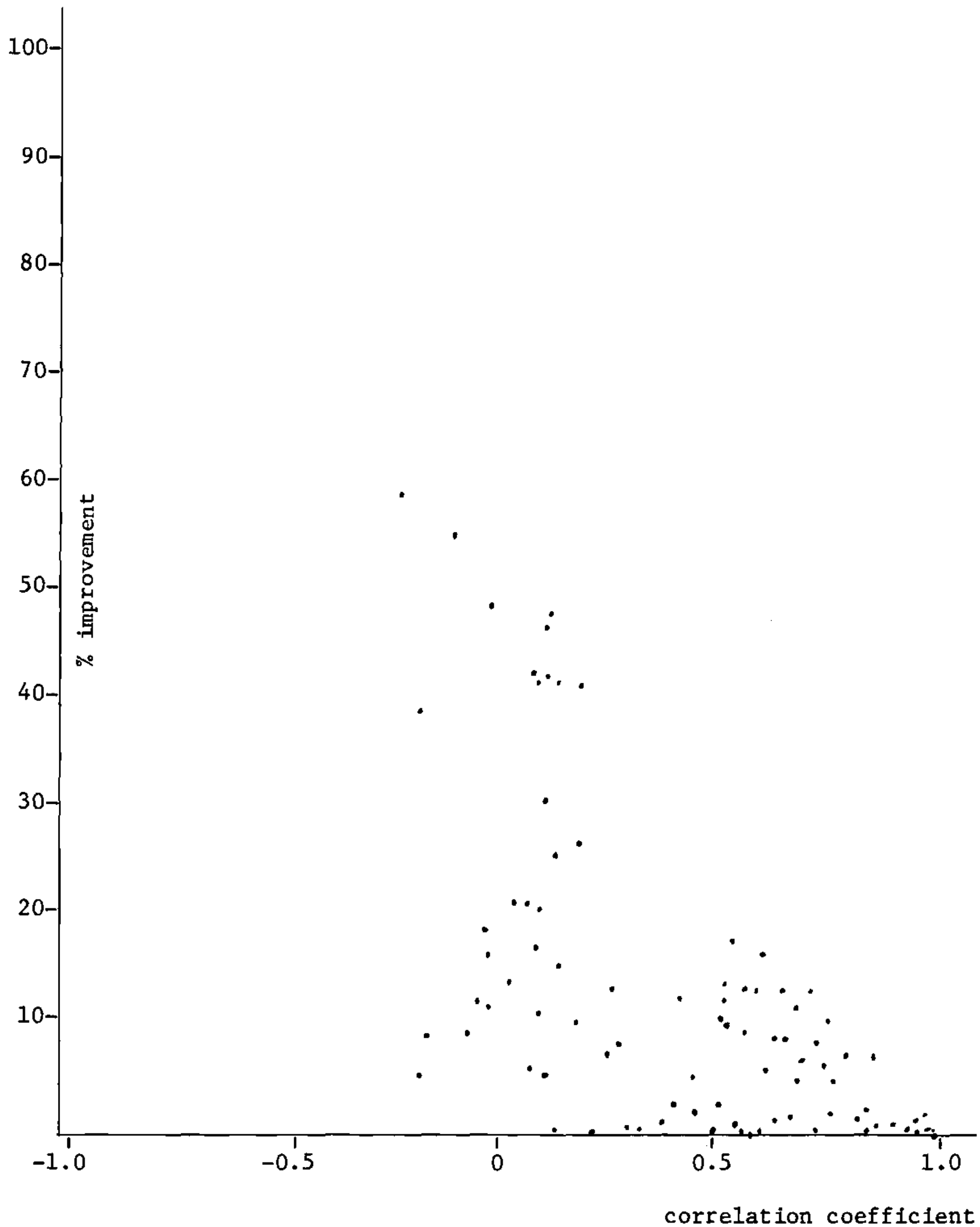


Figure 5.1. Percent of Improvement as a Function of Correlation Coefficient for the Combination of two Forecasting Methods.

Table 5.9 Comparative Analysis: Best Forecasting Method versus Best Combination of Two Methods

Series Used In Combination	Best Individual Forecasting Method				Best Combination			
	Method		Fitting	Forecasting	Method and Weights		Fitting*	Forecasting
4	Smoothing	$\sum e$	1.0472	0.3680	W-S	$\sum e$	-1.8177	-0.1163
		$\bar{e}$	0.0068	0.0023	W_w = 0.2671	$\bar{e}$	-0.0121	-0.0008
		v(e)	0.0833	0.1129	W_s = 0.7329	v(e)	0.3198	0.0172
		σ_e	0.29	0.3360	α = 0.80	σ_e	0.5655	0.1313
		MSE	0.0833	0.1121		MSE	0.3199	0.0172
9	Winters	$\sum e$	26.5071	-0.5466	BJ-Winters	$\sum e$	25.7392	-0.7449
		$\bar{e}$	0.2346	-0.0048	W_{BJ} = 0.5942	$\bar{e}$	0.2278	-0.0066
		v(e)	5.9230	0.0125	W_w = 0.4058	v(e)	2.6820	0.0032
		σ_e	2.4337	0.1120	α = 0.80	σ_e	1.6377	0.0564
		MSE	0.0126	0.0124		MSE	2.7404	0.0032
13	Winters (100 points used)	$\sum e$	5.2904	16.6793	A-W	$\sum e$	-710.4049	10.3862
		$\bar{e}$	0.0529	0.2034	W_A = 0.5101	$\bar{e}$	-8.6635	0.1267
		v(e)	381.7817	11.6363	W_w = 0.4899	v(e)	2071.85	2.2944
		σ_e	19.5392	3.4112	(98 points used)	σ_e	45.5177	1.5147
		MSE	9.5548	11.5358	α = 0.80	MSE	2130.84	2.3106
14	Winters	$\sum e$	-18.3095	-4.6250	A-R	$\sum e$	8.8742	-8.4023
		$\bar{e}$	-0.2441	-0.1850	W_A = 1.027	$\bar{e}$	0.3550	-0.3361
		v(e)	44.1026	13.8010	W_R = -0.027	v(e)	8.5066	1.5420
		σ_e	6.6410	3.7150	α = 0.80	σ_e	2.9166	1.2418
		MSE	4.4876	13.2832		MSE	8.5233	1.6596
15	Winters	$\sum e$	216.85	10.03	BJ-W	$\sum e$	50.2299	
		$\bar{e}$	2.9304	0.4010	W_B = 0.5332	$\bar{e}$	2.0092	0.5992
		v(e)	664.7204	2.0002	W_w = 0.4668	v(e)	21.7106	1.3484
		σ_e	25.7822	1.4143		σ_e	4.6595	1.1612
		MSE	0.7030	2.0809	α = 0.80	MSE	22.2484	1.7225

*Modeling phase with no updating of weights

Continuation of Table 5.9

Series Used In Combination	Best Individual Forecasting Method				Best Combination			
	Method		Fitting	Forecasting	Method and Weights		Fitting	Forecasting
16	Winters	$\sum e$	-0.2653	0.1812	W-R	$\sum e$	-0.6236	0.2011
		$\bar{e}$	-0.0035	0.0072	W_W=0.7817	$\bar{e}$	-0.0249	0.0080
		v(e)	0.0271	0.0232	W_R=0.2183	v(e)	0.0243	0.0020
		σ_e	0.1646	0.1524		σ_e	0.1560	0.0452
		MSE	0.0076	0.0223	α=0.80	MSE	0.0244	0.0021
17	Winters	$\sum e$	-2.9742	0.9961	W-R	$\sum e$	8.3951	-1.7975
		$\bar{e}$	-0.0397	0.0386	W_W=0.7566	$\bar{e}$	0.3358	0.0719
		v(e)	0.5869	0.2074	W_R=0.2434	v(e)	1.8062	0.0715
		σ_e	0.7661	0.4554		σ_e	1.3439	0.2674
		MSE	0.0678	0.2006	α=0.80	MSE	1.8212	0.0768
18	Winters	$\sum e$	-0.5570	0.5018	W-R	$\sum e$	-0.4264	0.5541
		$\bar{e}$	-0.0074	0.0201	W_W=0.8089	$\bar{e}$	-0.0171	0.0222
		v(e)	0.0619	0.0774	W_R=0.1911	v(e)	0.0539	0.0056
		σ_e	0.2488	0.2782		σ_e	0.2321	0.0749
		MSE	0.0252	0.0747	α=0.80	MSE	0.0539	0.0612
19	Winters	$\sum e$	17.6905	-0.4218	W-S	$\sum e$	2.3123	-0.0627
		$\bar{e}$	0.1769	-0.0043	W_W=0.4132	$\bar{e}$	0.0238	-0.0006
		v(e)	3.1419	0.0984	W_S=0.5868	v(e)	0.0950	0.0196
		σ_e	1.7725	0.3137		σ_e	0.3082	0.1399
		MSE	0.0954	0.0974	α=0.80	MSE	0.0956	0.0196
20	Adaptive	$\sum e$	2.1546	3.8610	A-W	$\sum e$	0.1074	1.7381
		$\bar{e}$	-0.0342	0.1287	W_A=0.4460	$\bar{e}$	0.0036	0.0579
		v(e)	0.3872	1.1092	W_W=0.5540	v(e)	0.3526	0.0479
		σ_e	0.6223	1.0532		σ_e	0.5938	0.2188
		MSE	0.4487	1.7192	α=0.80	MSE	0.3526	0.0513

Filtering and Box-Jenkins yields a high positive correlation coefficient between their forecast errors, resulting in small improvements when a combination between these two methods is performed. Combinations including Winters' Method perform very well, with improvements as high as 57.38%.

Combinations of methods involving Adaptive Filtering, Winters' Method or Smoothing Techniques are very useful, and the results of this combination outperform Box-Jenkins. The best combination of two methods for a given time series always outperforms the best individual forecasting method for its forecasting phase (Table 5.9).

5.3 Effect of Increasing the Number of Forecasting Methods in the Combination

The correlation coefficients between the errors of the different forecasting methods can be used as an indication of the possibilities of the combination. The same relationship seen for the combination of two methods between the correlation coefficient and percent of improvement holds true when combining three methods. When the correlation coefficient is small and negative, the percent of improvement is high (61.38 as an example). For the same series, combinations of three forecasting methods gives better results than combinations of only two, since more information is being used to forecast the time series. Series 9, Table 5.8, shows that a combination of Adaptive Filtering with Box-Jenkins gives a percent of improvement of 0.29%. The addition of Winters' Method to the combination increases the percent of improvement to 34.22% (Table 5.10).

In general, an increase in the number of forecasting methods

Table 5.10 Results for the Combination of Three Forecasting Methods

Series		A-B-W		A-B-R		A-B-S		A-W-R		A-W-S		A-R-S		B-W-R		B-W-S		B-R-S		W-S-R
4	ρ_{AB}	0.9738	ρ_{AB}	0.9785	ρ_{AB}	0.9781	ρ_{AW}	0.5638	ρ_{AW}	0.5874	ρ_{AR}	0.2518	ρ_{BW}	0.5642	ρ_{BW}	0.5949	ρ_{BR}	0.3517	ρ_{WS}	X
	ρ_{AW}	0.5652	ρ_{AR}	0.2573	ρ_{AS}	0.9719	ρ_{AR}	0.2583	ρ_{AS}	0.9720	ρ_{AS}	0.9756	ρ_{BR}	0.3583	ρ_{BS}	0.9986	ρ_{BS}	1.0000	ρ_{WR}	
	ρ_{BW}	0.5676	ρ_{BR}	0.3545	ρ_{BS}	0.9947	ρ_{WR}	0.1552	ρ_{WS}	0.5964	ρ_{RS}	0.3728	ρ_{WR}	0.1547	ρ_{WS}	0.5964	ρ_{RS}	0.3728	ρ_{SR}	
	σ_e^2	0.0784	σ_e^2	0.0789	σ_e^2	0.0841	σ_e^2	0.0724	σ_e^2	0.0782	σ_e^2	0.0774	σ_e^2	0.0731	σ_e^2	0.0780	σ_e^2	0.0802	σ_e^2	
	%	8.44	%	7.86	%	0.20	%	19.95	%	7.34	%	8.23	%	14.57	%	7.37	%	4.83	%	
9	ρ_{AB}	0.2948	ρ_{AB}	0.8885	ρ_{AB}	0.9149	ρ_{AW}	0.0696	ρ_{AW}	0.0419	ρ_{AR}	-0.0954	ρ_{BW}	0.0225	ρ_{BW}	0.0233	ρ_{BR}	-0.1632	ρ_{WS}	-0.0396
	ρ_{AW}	0.1979	ρ_{AR}	-0.1551	ρ_{AS}	-0.0938	ρ_{AR}	-0.1551	ρ_{AS}	-0.1005	ρ_{AS}	-0.1571	ρ_{BR}	-0.1785	ρ_{BS}	0.2111	ρ_{BS}	0.2162	ρ_{WR}	-0.0040
	ρ_{BW}	0.0127	ρ_{BR}	-0.1790	ρ_{BS}	0.2141	ρ_{WR}	-0.0125	ρ_{WS}	-0.0396	ρ_{RS}	0.0993	ρ_{WR}	-0.0125	ρ_{WS}	-0.0374	ρ_{RS}	0.0993	ρ_{SR}	0.0993
	σ_e^2	0.0239	σ_e^2	0.0234	σ_e^2	0.0255	σ_e^2	0.0169	σ_e^2	0.0156	σ_e^2	0.0247	σ_e^2	0.0146	σ_e^2	0.0162	σ_e^2	0.0241	σ_e^2	0.0340
	%	34.22	%	6.24	%	1.57	%	42.52	%	45.24	%	13.23	%	41.50	%	36.39	%	5.32	%	9.00
14	ρ_{AB}	0.6802	ρ_{AB}	0.6942	ρ_{AB}	0.6883	ρ_{AW}	0.5202	ρ_{AW}	0.4920	ρ_{AR}	0.1250	ρ_{BW}	0.5136	ρ_{BW}	0.5318	ρ_{BR}	0.1894	ρ_{WS}	0.3790
	ρ_{AW}	0.5152	ρ_{AR}	0.1547	ρ_{AS}	0.6594	ρ_{AR}	0.1508	ρ_{AS}	0.6593	ρ_{AS}	0.6661	ρ_{BR}	0.2079	ρ_{BS}	0.4670	ρ_{BS}	0.4697	ρ_{WR}	0.1001
	ρ_{BW}	0.5485	ρ_{BR}	0.2013	ρ_{BS}	0.4628	ρ_{WR}	0.1166	ρ_{WS}	0.3729	ρ_{RS}	0.5956	ρ_{WR}	0.1153	ρ_{WS}	0.3729	ρ_{RS}	0.5957	ρ_{SR}	0.5957
	σ_e^2	10.9844	σ_e^2	8.4996	σ_e^2	9.2423	σ_e^2	8.4444	σ_e^2	9.2935	σ_e^2	8.2706	σ_e^2	12.7448	σ_e^2	10.4987	σ_e^2	10.2606	σ_e^2	11.6075
	%	2.88	%	21.33	%	12.38	%	21.77	%	11.89	%	21.56	%	34.36	%	13.25	%	15.21	%	4.08
15	ρ_{AB}	0.8415	ρ_{AB}	0.8414	ρ_{AB}	0.8760	ρ_{AW}	-0.0405	ρ_{AW}	-0.0587	ρ_{AR}	-0.0198	ρ_{BW}	-0.1743	ρ_{BW}	0.6170	ρ_{BR}	0.6082	ρ_{WS}	-0.2761
	ρ_{AW}	-0.0401	ρ_{AR}	-0.0139	ρ_{AS}	0.3046	ρ_{AR}	-0.0164	ρ_{AS}	-0.3086	ρ_{AS}	0.3099	ρ_{BR}	0.1620	ρ_{BS}	-0.1800	ρ_{BS}	0.1587	ρ_{WR}	-0.1628
	ρ_{BW}	-0.1875	ρ_{BR}	0.1587	ρ_{BS}	0.6210	ρ_{WR}	-0.1620	ρ_{WS}	-0.2732	ρ_{RS}	0.4856	ρ_{WR}	-0.1561	ρ_{WS}	-0.2732	ρ_{RS}	0.4774	ρ_{SR}	0.4843
	σ_e^2	2.8049	σ_e^2	6.0993	σ_e^2	6.6592	σ_e^2	3.2571	σ_e^2	3.0188	σ_e^2	6.5745	σ_e^2	2.7828	σ_e^2	2.8708	σ_e^2	6.3343	σ_e^2	3.8626
	%	56.88	%	6.29	%	0.75	%	57.25	%	61.38	%	19.44	%	56.48	%	57.11	%	5,74	%	50.59
16	ρ_{AB}	0.7860	ρ_{AB}	0.7773	ρ_{AB}	0.7745	ρ_{AW}	0.9088	ρ_{AW}	0.9070	ρ_{AR}	0.1343	ρ_{BW}	0.7936	ρ_{BW}	0.7628	ρ_{BR}	0.4325	ρ_{WS}	0.6406
	ρ_{AW}	0.9087	ρ_{AR}	0.1503	ρ_{AS}	0.6552	ρ_{AR}	0.1496	ρ_{AS}	0.6397	ρ_{AS}	0.6639	ρ_{BR}	0.4391	ρ_{BS}	0.9609	ρ_{BS}	0.9624	ρ_{WR}	0.1009
	ρ_{BW}	0.7790	ρ_{BR}	0.4411	ρ_{BS}	0.9503	ρ_{WR}	0.1085	ρ_{WS}	0.6404	ρ_{RS}	0.5460	ρ_{WR}	0.1101	ρ_{WS}	0.6474	ρ_{RS}	0.5457	ρ_{SR}	0.5362
	σ_e^2	0.0268	σ_e^2	0.0272	σ_e^2	0.0282	σ_e^2	0.0239	σ_e^2	0.0261	σ_e^2	0.0266	σ_e^2	0.0234	σ_e^2	0.0262	σ_e^2	0.0286	σ_e^2	0.0235
	%	7.20	%	14.48	%	9.57	%	17.10	%	8.05	%	18.63	%	16.99	%	9.27	%	8.52	%	17.16

An X means combination not performed.

Continuation of Table 5.10

Series		A-B-W		A-B-R		A-B-S		A-W-R		A-W-S		A-R-S		B-W-R		B-W-S		B-R-S		W-S-R
17	ρ_{AB}	0.8682	ρ_{AB}	0.8825	ρ_{AB}	0.8826	ρ_{AW}	0.7995	ρ_{AW}	0.7996	ρ_{AR}	0.2780	ρ_{BW}	0.7866	ρ_{BW}	0.7984	ρ_{BR}	0.2850	ρ_{WS}	0.1069
	ρ_{AW}	0.8147	ρ_{AR}	0.2787	ρ_{AS}	0.7970	ρ_{AR}	0.2804	ρ_{AS}	0.7970	ρ_{AS}	0.8032	ρ_{BR}	0.2806	ρ_{BS}	0.6820	ρ_{BS}	0.6865	ρ_{WR}	0.5615
	ρ_{BW}	0.7994	ρ_{BR}	0.2783	ρ_{BS}	0.6766	ρ_{WR}	0.1035	ρ_{WS}	0.5515	ρ_{RS}	0.6532	ρ_{WR}	0.0985	ρ_{WS}	0.5515	ρ_{RS}	0.6532	ρ_{SR}	0.6502
	σ_e^2	0.5257	σ_e^2	0.5072	σ_e^2	0.5778	σ_e^2	0.4864	σ_e^2	0.5906	σ_e^2	0.5705	σ_e^2	0.4626	σ_e^2	0.5463	σ_e^2	0.5332	σ_e^2	0.5195
	%	6.76	%	9.12	%	1.91	%	21.39	%	9.90	%	18.65	%	16.47	%	7.25	%	9.55	%	20.69
18	ρ_{AB}	0.7200	ρ_{AB}	0.7302	ρ_{AB}	X	ρ_{AW}	0.7857	ρ_{AW}	X	ρ_{AR}	X	ρ_{BW}	0.7239	ρ_{BW}	X	ρ_{BR}	X	ρ_{WS}	X
	ρ_{AW}	0.7962	ρ_{AR}	0.1275	ρ_{AS}		ρ_{AR}	0.1232	ρ_{AS}		ρ_{AS}		ρ_{BR}	0.5553	ρ_{BS}		ρ_{BS}		ρ_{WR}	
	ρ_{BW}	0.7091	ρ_{BR}	0.5615	ρ_{BS}		ρ_{WR}	0.1317	ρ_{WS}		ρ_{RS}		ρ_{WR}	0.1332	ρ_{WS}		ρ_{RS}		ρ_{SR}	
	σ_e^2	0.5750	σ_e^2	0.0626	σ_e^2		σ_e^2	0.0544	σ_e^2		σ_e^2		σ_e^2	0.0548	σ_e^2		σ_e^2		σ_e^2	
	%	10.31	%	9.81	%		%	16.58	%		%		%	15.03	%		%		%	
19	ρ_{AB}	0.9906	ρ_{AB}	0.9895	ρ_{AB}	0.9883	ρ_{AW}	0.0863	ρ_{AW}	0.1040	ρ_{AR}	0.6445	ρ_{BW}	0.0867	ρ_{BW}	0.1039	ρ_{BR}	0.6446	ρ_{WS}	0.0381
	ρ_{AW}	0.0940	ρ_{AR}	0.5977	ρ_{AS}	0.9211	ρ_{AR}	0.6050	ρ_{AS}	0.9235	ρ_{AS}	0.9236	ρ_{BR}	0.6050	ρ_{BS}	0.9215	ρ_{BS}	0.9239	ρ_{WR}	-0.0587
	ρ_{BW}	0.0872	ρ_{BR}	0.5968	ρ_{BS}	0.9135	ρ_{WR}	-0.0813	ρ_{WS}	0.0278	ρ_{RS}	0.7363	ρ_{WR}	-0.0813	ρ_{WS}	0.0278	ρ_{RS}	0.7363	ρ_{SR}	0.7362
	σ_e^2	0.0687	σ_e^2	0.1144	σ_e^2	0.1117	σ_e^2	0.0587	σ_e^2	0.0598	σ_e^2	0.1030	σ_e^2	0.0587	σ_e^2	0.0598	σ_e^2	0.1030	σ_e^2	0.0561
	%	42.22	%	15.38	%	0.39	%	51.21	%	46.69	%	8.16	%	51.20	%	46.66	%	8.14	%	49.99
13	ρ_{AB}	X	ρ_{AB}	X	ρ_{AB}	X	ρ_{AW}	X	ρ_{AW}	0.0221	ρ_{AR}	X	ρ_{BW}	X	ρ_{BW}	X	ρ_{BR}	X	ρ_{WS}	X
	ρ_{AW}		ρ_{AR}		ρ_{AS}		ρ_{AR}		ρ_{AS}	0.9546	ρ_{AS}		ρ_{BR}		ρ_{BS}		ρ_{BS}		ρ_{WR}	
	ρ_{BW}		ρ_{BR}		ρ_{BS}		ρ_{WR}		ρ_{WS}	0.0183	ρ_{RS}		ρ_{WR}		ρ_{WS}		ρ_{RS}		ρ_{SR}	
	σ_e^2		σ_e^2		σ_e^2		σ_e^2		σ_e^2	147.86	σ_e^2		σ_e^2		σ_e^2		σ_e^2		σ_e^2	
	%		%		%		%		%	47.46	%		%		%		%		%	
20	ρ_{AB}	0.6387	ρ_{AB}	0.7087	ρ_{AB}	0.7101	ρ_{AW}	0.6912	ρ_{AW}	0.6921	ρ_{AR}	0.3060	ρ_{BW}	0.6757	ρ_{BW}	0.7381	ρ_{BR}	X	ρ_{WS}	0.4724
	ρ_{AW}	0.7343	ρ_{AR}	0.3095	ρ_{AS}	0.2707	ρ_{AR}	0.3115	ρ_{AS}	0.2707	ρ_{AS}	0.2708	ρ_{BR}	0.5767	ρ_{BS}	0.5575	ρ_{BS}		ρ_{WR}	0.5393
	ρ_{BW}	0.6703	ρ_{BR}	0.5701	ρ_{BS}	0.5495	ρ_{WR}	0.5717	ρ_{WS}	0.6302	ρ_{RS}	0.8716	ρ_{WR}	0.5227	ρ_{WS}	0.6292	ρ_{RS}		ρ_{SR}	0.8671
	σ_e^2	0.3945	σ_e^2	0.4047	σ_e^2	0.4025	σ_e^2	0.3643	σ_e^2	0.3425	σ_e^2	0.4768	σ_e^2	0.3822	σ_e^2	0.3235	σ_e^2		σ_e^2	0.3986
	%	11.76	%	15.23	%	15.69	%	20.05	%	24.66	%	0.13	%	26.94	%	29.06	%		%	12.90

An X means combination not performed

Table 5.11 Combinations of Four Forecasting Methods

Series	A-B-W-R		A-B-W-S		A-W-R-S		B-W-R-S		A-B-S-R	
4	ρ_{AB}	0.9725	ρ_{AB}	0.9781	ρ_{AW}	0.5882	ρ_{BW}	X	ρ_{AB}	X
	ρ_{AW}	0.5689	ρ_{AW}	0.5913	ρ_{AR}	0.2547	ρ_{BR}		ρ_{AS}	
	ρ_{AR}	0.2570	ρ_{AS}	0.9719	ρ_{AS}	0.9756	ρ_{BS}		ρ_{AR}	
	ρ_{BW}	0.5633	ρ_{BW}	0.5949	ρ_{WR}	0.1557	ρ_{WR}		ρ_{BS}	
	ρ_{BR}	0.3526	ρ_{BS}	0.9985	ρ_{WS}	0.6025	ρ_{WS}		ρ_{BR}	
	ρ_{WR}	0.1552	ρ_{WS}	0.5964	ρ_{RS}	0.3728	ρ_{RS}		ρ_{SR}	
	σ_e^2	0.0725	σ_e^2	0.0780	σ_e^2	0.0713	σ_e^2		σ_e^2	
	%	16.24	%	7.46	%	15.56	%		%	
9	ρ_{AB}	0.8879	ρ_{AB}	0.8561	ρ_{AW}	0.0419	ρ_{BW}	0.0248	ρ_{AB}	0.9326
	ρ_{AW}	0.0695	ρ_{AW}	0.0270	ρ_{AR}	-0.1570	ρ_{BR}	-0.1629	ρ_{AS}	-0.0953
	ρ_{AR}	-0.1551	ρ_{AS}	-0.0958	ρ_{AS}	-0.0959	ρ_{BS}	0.2142	ρ_{AR}	-0.1571
	ρ_{BW}	0.0224	ρ_{BW}	0.0149	ρ_{WR}	-0.0038	ρ_{WR}	-0.0038	ρ_{BS}	0.2162
	ρ_{BR}	-0.1771	ρ_{BS}	0.6011	ρ_{WS}	-0.0394	ρ_{WS}	-0.0394	ρ_{BR}	-0.1632
	ρ_{WR}	-0.0124	ρ_{WS}	-0.0259	ρ_{RS}	-0.0017	ρ_{RS}	0.0989	ρ_{SR}	0.0993
	σ_e^2	0.0147	σ_e^2	0.0211	σ_e^2	0.0153	σ_e^2	0.0150	σ_e^2	0.0240
	%	41.27	%	25.93	%	46.29	%	41.06	%	5.40
14	ρ_{AB}	0.6768	ρ_{AB}	0.6880	ρ_{AW}	0.4943	ρ_{BW}	0.5362	ρ_{AB}	0.6884
	ρ_{AW}	0.5145	ρ_{AW}	0.4998	ρ_{AR}	0.1278	ρ_{BR}	0.1927	ρ_{AS}	0.6661
	ρ_{AR}	0.1438	ρ_{AS}	0.6590	ρ_{AS}	0.6662	ρ_{BS}	0.4718	ρ_{AR}	0.1351
	ρ_{BW}	0.5446	ρ_{BW}	0.5321	ρ_{WR}	0.1007	ρ_{WR}	0.1007	ρ_{BS}	0.4697
	ρ_{BR}	0.2052	ρ_{BS}	0.4671	ρ_{WS}	0.3791	ρ_{WS}	0.3791	ρ_{BR}	0.1894
	ρ_{WR}	0.1167	ρ_{WS}	0.3729	ρ_{RS}	0.5993	ρ_{RS}	0.5993	ρ_{SR}	0.5956
	σ_e^2	8.6401	σ_e^2	9.1967	σ_e^2	8.2510	σ_e^2	10.2491	σ_e^2	8.3294
	%	23.57	%	12.87	%	21.74	%	15.31	%	21.00
15	ρ_{AB}	0.8415	ρ_{AB}	0.8832	ρ_{AW}	-0.0576	ρ_{BW}	-0.1860	ρ_{AB}	0.8839
	ρ_{AW}	-0.0137	ρ_{AW}	-0.0582	ρ_{AR}	-0.0197	ρ_{BR}	0.1576	ρ_{AS}	0.3156
	ρ_{AR}	-0.0405	ρ_{AS}	0.3104	ρ_{AS}	0.3156	ρ_{BS}	0.6187	ρ_{AR}	-0.0199
	ρ_{BW}	0.1582	ρ_{BW}	-0.1843	ρ_{WR}	-0.1610	ρ_{WR}	-0.1620	ρ_{BS}	0.6312
	ρ_{BR}	-0.1901	ρ_{BS}	0.6256	ρ_{WS}	-0.2649	ρ_{WS}	-0.2762	ρ_{BR}	0.1591
	ρ_{WR}	-0.1620	ρ_{WS}	-0.2732	ρ_{RS}	0.4835	ρ_{RS}	0.4821	ρ_{SR}	0.4865
	σ_e^2	2.6599	σ_e^2	2.8775	σ_e^2	2.9569	σ_e^2	2.7549	σ_e^2	6.2513
	%	59.13	%	57.04	%	62.65	%	59.00	%	6.60

An X means combination not performed.

Continuation of Table 5.11

Series	A-B-W-R		A-B-W-S		A-W-R-S		B-W-R-S		A-B-S-R	
16	ρ_{AB}	0.7859	ρ_{AB}	0.7746	ρ_{AW}	0.8861	ρ_{BW}	0.7822	ρ_{AB}	0.7633
	ρ_{AW}	0.9087	ρ_{AW}	0.9070	ρ_{AR}	0.1358	ρ_{BR}	0.4327	ρ_{AS}	0.6566
	ρ_{AR}	0.1515	ρ_{AS}	0.6433	ρ_{AS}	0.6617	ρ_{BS}	0.9528	ρ_{AR}	0.1339
	ρ_{BW}	0.7944	ρ_{BW}	0.7806	ρ_{WR}	0.0929	ρ_{WR}	0.1009	ρ_{BS}	0.9554
	ρ_{BR}	0.4375	ρ_{BS}	0.9329	ρ_{WS}	0.6620	ρ_{WS}	0.6474	ρ_{BR}	0.4323
	ρ_{WR}	0.1085	ρ_{WS}	0.6404	ρ_{RS}	0.5433	ρ_{RS}	0.5418	ρ_{SR}	0.5461
	σ^2_e	0.0239	σ^2_e	0.0261	σ^2_e	0.0237	σ^2_e	0.0235	σ^2_e	0.0263
	%	17.10	%	8.10	%	18.00	%	17.17	%	16.07
17	ρ_{AB}	0.8696	ρ_{AB}	0.8824	ρ_{AW}		ρ_{BW}	0.7973	ρ_{AB}	0.8819
	ρ_{AW}	0.8062	ρ_{AW}	0.8110	ρ_{AR}		ρ_{BR}	0.2879	ρ_{AS}	0.2854
	ρ_{AR}	0.2733	ρ_{AS}	0.7969	ρ_{AS}		ρ_{BS}	0.6885	ρ_{AR}	0.8032
	ρ_{BW}	0.7829	ρ_{BW}	0.7982	ρ_{WR}		ρ_{WR}	0.1069	ρ_{BS}	0.2850
	ρ_{BR}	0.2789	ρ_{BS}	0.6819	ρ_{WS}		ρ_{WS}	0.5615	ρ_{BR}	0.6865
	ρ_{WR}	0.1031	ρ_{WS}	0.5515	ρ_{RS}		ρ_{RS}	0.6502	ρ_{SR}	0.6532
	σ^2_e	0.4675	σ^2_e	0.5386	σ^2_e		σ^2_e	0.4815	σ^2_e	0.5197
	%	18.68	%	8.61	%		%	18.55	%	11.86
18	ρ_{AB}	0.7315	ρ_{AB}		ρ_{AW}		ρ_{BW}		ρ_{AB}	
	ρ_{AW}	0.7870	ρ_{AW}		ρ_{AR}		ρ_{BR}		ρ_{AS}	
	ρ_{AR}	0.1267	ρ_{AS}		ρ_{AS}		ρ_{BS}		ρ_{AR}	
	ρ_{BW}	0.7033	ρ_{BW}		ρ_{WR}		ρ_{WR}		ρ_{BS}	
	ρ_{BR}	0.5759	ρ_{BS}		ρ_{WS}		ρ_{WS}		ρ_{BR}	
	ρ_{WR}	0.1295	ρ_{WS}		ρ_{RS}		ρ_{RS}		ρ_{SR}	
	σ^2_e	0.0553	σ^2_e		σ^2_e		σ^2_e		σ^2_e	
	%	17.93	%		%		%		%	
19	ρ_{AB}	0.9901	ρ_{AB}	0.9984	ρ_{AW}	0.1040	ρ_{BW}		ρ_{AB}	0.9889
	ρ_{AW}	0.0933	ρ_{AW}	0.1050	ρ_{AR}	0.6515	ρ_{BR}		ρ_{AS}	0.9235
	ρ_{AR}	0.5977	ρ_{AS}	0.9211	ρ_{AS}	0.9236	ρ_{BS}		ρ_{AR}	0.6456
	ρ_{BW}	0.0866	ρ_{BW}	0.1039	ρ_{WR}	-0.0587	ρ_{WR}		ρ_{BS}	0.9239
	ρ_{BR}	0.6040	ρ_{BS}	0.9215	ρ_{WS}	0.0381	ρ_{WS}		ρ_{BR}	0.6446
	ρ_{WR}	-0.0812	ρ_{WS}	0.0278	ρ_{RS}	0.7362	ρ_{RS}		ρ_{SR}	0.7363
	σ^2_e	0.0587	σ^2_e	0.0598	σ^2_e	0.0558	σ^2_e		σ^2_e	0.1030
	%	51.28	%	46.69	%	50.23	%		%	8.16

An X means combination not performed.

Continuation of Table 5.11

Series	A-B-W-R		A-B-W-S		A-W-R-S		B-W-R-S		A-B-S-R	
20	ρ_{AB}	0.6585	ρ_{AB}	0.7092	ρ_{AW}		ρ_{BW}	0.7390	ρ_{AB}	0.7101
	ρ_{AW}	0.6453	ρ_{AW}	0.7025	ρ_{AR}		ρ_{BR}	0.5718	ρ_{AS}	0.2708
	ρ_{AR}	0.3137	ρ_{AS}	0.2707	ρ_{AS}		ρ_{BS}	0.5542	ρ_{AR}	0.3092
	ρ_{BW}	0.6068	ρ_{BW}	0.7367	ρ_{WR}		ρ_{WR}	0.5671	ρ_{BS}	0.5587
	ρ_{BR}	0.5453	ρ_{BS}	0.5569	ρ_{WS}		ρ_{WS}	0.6393	ρ_{BR}	0.5709
	ρ_{WR}	0.5068	ρ_{WS}	0.6288	ρ_{RS}		ρ_{RS}	0.8651	ρ_{SR}	0.8716
	σ_e^2	0.3823	σ_e^2	0.3231	σ_e^2		σ_e^2	0.3203	σ_e^2	0.3999
	%	19.86	%	29.24	%		%	29.71	%	16.23

An X means combination not performed.

Table 5.12 Combination of Five Forecasting Methods (A-B-W-R-S)

Series	ρ_{AB}	ρ_{AW}	ρ_{AR}	ρ_{AS}	ρ_{BW}	ρ_{BR}	ρ_{BS}	ρ_{WR}	ρ_{WS}	ρ_{RS}	σ_e	%
4	0.9767	0.5882	0.2558	0.9739	0.5924	0.3543	0.9932	0.1552	0.5959	0.3730	0.0714	15.33
9	0.9310	0.0463	-0.1556	-0.0957	0.0233	-0.1612	0.2058	0.0021	-0.0356	0.0948	0.0157	38.43
14	0.6772	0.4939	0.1321	0.6730	0.5281	0.1954	0.4576	0.1010	0.3798	0.6017	8.3169	21.97
15	0.8755	-0.0458	-0.0206	0.3085	-0.1619	0.1547	0.6025	-0.1630	-0.2704	0.4765	2.8412	58.46
16	0.7640	0.1431	0.6583	0.9051	0.4264	0.9336	0.7613	0.5426	0.0935	0.6486	0.0234	17.65
17	0.8821	0.7964	0.2850	0.8013	0.7873	0.2909	0.6732	0.1057	0.5555	0.6523	0.4852	17.84
19	0.9821	0.1113	0.6543	0.9146	0.1039	0.6455	0.9125	-0.0588	0.0292	0.7367	0.0552	50.74
20	0.7103	0.6634	0.3136	0.2691	0.6750	0.5768	0.5520	0.5160	0.6007	0.8622	0.3486	27.10

involved in the combination improves the variance of the forecasting phase Tables 5.11 and 5.12. However, the concept of parsimony should be considered when increasing the number of methods. A parsimonious model is one that adequately describes the series of data but requires as few parameters as possible. Increasing the number of methods in the combination increases the number of parameters. In fact, even when the variance of the forecast errors for the combination decreases, the new mean square error may tend to get larger as a result of a reduction in degrees of freedom.

5.4 Comparative Analysis: Bates and Granger Approach Versus the Proposed Method

Bates and Granger [14] have suggested various heuristic techniques for the combination of forecasts. Their techniques were compared to the approach of combining forecasts by the use of some statistical measures. Results of this comparison analysis are tabulated in Tables 5.13 and 5.14. Formulas (10) and (14) were used for the comparative phase. Table 5.13 considers only two forecasting methods, while Table 5.14 assumes that all available forecasting methods were combined.

Since Bates and Granger [14] initially used the mean or average of all forecasts as a method of combining forecasts, a computation for the average is also considered, where the average forecast for period t is

$$\text{Average}_t = \frac{\hat{X}_{1,t} + \hat{X}_{2,t} + \cdots + \hat{X}_{n,t}}{n}$$

where $\hat{X}_{i,t}$ is the forecast obtained from method i for period t. It is evident from examining Tables 5.13 and 5.14 that the proposed method for the combination of forecasts performs much better than any of the heuristic methods mentioned by Bates and Granger.

Several runs were made changing the parameters of the individual formulas. We conclude that formula 2, which considers an estimate of $\sum$, performs best of all of Bates and Granger's formulas. Their previous results [14] indicate that Formula 1 usually performs best.

5.5 Updating the Weights for the Combination of Forecasts

The updating procedure for re-estimation of the elements in the covariance matrix, according to the formula

$$\hat{\sigma}_{ij}^{2}(t+1) = \alpha[e_{i,t+1}(t)\; e_{j,t+1}(t)] + (1 - \alpha)\; \hat{\alpha}_{ij}^{2}(t).$$

Changing the value of the smoothing constant gives some interesting results. Table 5.15 presents the results for the forecasting phase of the combination of two methods using smoothing constants of 0.2, 0.4, 0.6 and 0.8. The results for the "best" combination of methods, changing the smoothing constant, also varied over the range from 0.2 to 0.8 are summarized in Table 5.16.

From studying Tables 5.15 and 5.16 we note that when the smoothing constant (α) increases, the variance of the combined forecast error for two forecasting methods always decreases, although it may reach a minimum point somewhere in the interval $0 < \alpha < 1$ for combinations of more than two methods. We recall that this variance was defined as

$$V[e_{T+\tau}(T)] = \underline{w}_T' \textstyle\sum_T^{-1} \underline{w}_T$$

Since we are updating the elements of $\sum_T$ using a smoothing technique, the period errors from the individual forecasting methods receive more weight than the variance or covariance of the previous period of time. This suggests that the use of a large α results in a system of weights that can adapt quickly to changes in the performance of the individual forecasting methods.

Table 5.13. Comparative Analysis: Bates and Granger Approach Versus a Statistical Approach to the Combination of Forecasts (Two Methods Only)

Series	Best Comb. of Two Forecasting Method	Best Smoothing Constant		Average	Formula 10 (ν=6)	Formula 11 (β=6)	Formula 12 (γ=0.80)	Formula 13 (Θ=1.40)	Formula 14 (ϕ=1.20)	Formula 27
4	W-S	0.80	$\bar{e}$	-0.0256	-0.0118	-0.0111	-0.0184	0.0256	-0.0271	-0.0008
			V(e)	0.1239	0.1580	0.1237	0.1206	0.1587	0.1280	0.0172
			σ_e	0.3520	0.3975	0.3517	0.3472	0.3984	0.3578	0.1313
			MSE	0.1245	0.1581	0.1238	0.1209	0.1594	0.1287	0.0172
9	B-W	0.80	$\bar{e}$	-0.0079	-0.0143	-0.0092	-0.0085	-0.0100	-0.0085	-0.0066
			V(e)	0.0112	0.0139	0.0112	0.0112	0.0113	0.0111	0.0032
			σ_e	0.1058	0.1179	0.1058	0.1059	0.1065	0.1055	0.0564
			MSE	0.0113	0.0141	0.0113	0.0114	0.0114	0.0112	0.0032
13	A-W	0.80	$\bar{e}$	-0.2986	0.1631	-0.0192	-0.0879	-0.6004	-0.3250	0.1267
			V(e)	10.4475	14.1055	10.4630	10.6241	11.4627	10.4879	2.2944
			σ_e	3.2323	3.7557	3.2347	3.2595	3.3857	3.2385	1.5147
			MSE	10.5378	14.1324	10.4634	10.6319	11.8276	10.5948	2.3106
14	A-R	0.80	$\bar{e}$	-7.8940	-0.2912	-5.1315	-6.0636	-3.5862	-6.7143	-0.3361
			V(e)	15.7980	32.1618	11.7169	12.3935	13.5766	11.3851	1.5420
			σ_e	3.9747	5.6711	3.4230	3.5204	3.6846	3.3742	1.2418
			MSE	80.7097	32.2502	39.1465	50.6929	26.9735	58.3453	1.6596

Continuation of Table 5.13.

Series	Best Comb. of Two Forecasting Method	Best Smoothing Constant		Average	Formula 10 (ν=6)	Formula 11 (β=6)	Formula 12 (γ=0.80)	Formula 13 (Θ=1.40)	Formula 14 (ϕ=1.20)	Formula 27
15	B-W	0.80	$\bar{e}$	0.1500	0.4218	0.3209	0.3032	0.0684	0.0487	0.5992
			V(e)	2.0629	1.9701	2.0424	1.9769	2.4674	2.5969	1.3484
			σ_e	1.4363	1.4036	1.4291	1.4060	1.5708	1.6115	1.1612
			MSE	2.0864	2.1554	2.1496	2.0727	2.4722	2.5996	1.7225
16	W-R	0.80	$\bar{e}$	0.3384	0.0113	0.0446	0.0811	0.1779	0.1943	0.0080
			V(e)	0.0242	0.0325	0.0229	0.0222	0.0184	0.0184	0.0020
			σ_e	0.1556	0.1804	0.1512	0.1489	0.1356	0.1356	0.0452
			MSE	0.1435	0.0327	0.0249	0.0290	0.0513	0.0577	0.0021
17	W-R	0.80	$\bar{e}$	-0.7440	-0.1345	-0.1501	-0.2727	-0.1637	-0.4795	-0.0719
			V(e)	0.0860	0.1674	0.1611	0.1479	0.1606	0.1163	0.0715
			σ_e	0.2933	0.4092	0.4014	0.3846	0.4007	0.3410	0.2674
			MSE	0.6626	0.1863	0.1846	0.2254	0.1885	0.3558	0.0769
18	W-R	0.80	$\bar{e}$	0.3301	0.0640	0.1351	0.1788	0.2084	0.2048	0.0222
			V(e)	0.0643	0.1113	0.0724	0.0621	0.0611	0.0600	0.0056
			σ_e	0.2535	0.3336	0.2691	0.2493	0.2472	0.2450	0.0749
			MSE	0.1778	0.1155	0.0914	0.0955	0.1063	0.1037	0.0061

Continuation of Table 5.13.

Series	Best Comb. of Two Forecasting Method	Best Smoothing Constant		Average	Formula 10 (ν=6)	Formula 11 (β=6)	Formula 12 (γ=0.80)	Formula 13 (θ=1.40)	Formula 14 (ϕ1.20)	Formula 27
19	W-S	0.80	$\bar{e}$	0.0097	-0.0008	0.0091	0.0098	0.0118	0.0111	-0.0006
			V(e)	0.0923	0.1193	0.0928	0.0925	0.0932	0.0919	0.0196
			σ_e	0.3038	0.3454	0.3046	0.3041	0.3053	0.3031	0.1399
			MSE	0.0924	0.1193	0.0929	0.0925	0.0934	0.0920	0.0196
20	A-W	0.80	$\bar{e}$	0.1520	0.1232	0.1159	0.1341	0.1262	0.1457	0.0579
			V(e)	0.3468	0.6572	0.3880	0.3582	1.3120	0.4335	0.0479
			σ_e	0.5889	0.8107	0.6229	0.5985	1.1454	0.6584	0.2188
			MSE	0.3707	0.6729	0.4019	0.3768	1.3285	0.4554	0.0514

Table 5.14. Comparative Analysis: Bates and Granger Approach Versus the Proposed Method for the Combination of Forecasts (More Than Two Methods)

Series	Best Comb. of Forecasting Methods	Best Smoothing Constant (α) (For Formula 27)		Average	Formula 10 (ν=6)	Formula 11 (β=6)	Formula 12 (γ=0.80)	Formula 13 (θ=1.40)	Formula 14 (ϕ=1.20)	Formula 27
4	A-W-R-S	0.40	$\bar{e}$	-0.1859	-0.0610	-0.0432	-0.0618	0.3942	-0.0843	0.0041
			σ_e^2	0.1370	0.9466	0.1152	0.1107	4.3163	0.1137	0.0136
			σ_e	0.3702	0.9729	0.3395	0.3327	2.0776	0.3373	0.1166
			MSE	0.1718	0.9504	0.1171	0.1145	4.4728	0.1209	0.0136
9	B-W-R	0.80	$\bar{e}$	-0.3334	0.0000	-0.0122	-0.0182	-0.0108	-0.0839	-0.0021
			σ_e^2	0.2686	0.0154	0.0110	0.0113	0.0107	0.0115	0.0002
			σ_e	0.5182	0.1240	0.1047	0.1062	0.1033	0.1072	0.0155
			MSE	0.3807	0.0154	0.0111	0.0116	0.0108	0.0127	0.0002
13	A-W	0.80	$\bar{e}$	-0.2986	0.1631	-0.0192	-0.0879	-0.6004	-0.3250	0.1267
			σ_e^2	10.4475	14.1055	10.4630	10.6241	11.4627	10.4879	2.2944
			σ_e	3.2323	3.7557	3.2357	3.2595	3.3857	3.2385	1.5147
			MSE	10.5378	14.1324	10.4634	10.6319	11.8276	10.5948	2.3106
14	A-R-S	0.80	$\bar{e}$	-4.1890	-3.3508	-1.5945	-2.0692	-12.8298	-3.9849	0.0210
			σ_e^2	9.2405	78.1984	10.5765	9.8887	38.0460	8.8435	0.1816
			σ_e	3.0398	8.8430	3.2522	3.1446	6.1681	2.9738	0.4261
			MSE	27.5194	89.8939	13.2250	14.3485	209.5087	25.3845	0.1820

Continuation of Table 5.14.

Series	Best Comb. of Forecasting Methods	Best Smoothing Constant (α) (For Formula 27)		Average	Formula 10 (ν=6)	Formula 11 (β=6)	Formula 12 (γ=0.80)	Formula 13 (Θ=1.40)	Formula 14 (ϕ=1.20)	Formula 27
15	B-W-R-S	0.60	$\bar{e}$	2.4630	-0.6649	0.6645	0.9327	2.4049	1.2103	-0.1879
			σ_e^2	4.3763	6.1935	2.1361	1.7930	3.8936	2.6708	1.2594
			σ_e	2.0920	2.4887	1.4615	1.3390	1.9732	1.6343	1.1222
			MSE	10.6954	6.6540	2.5961	2.6991	9.9180	4.1967	1.2962
16	W-R	0.80	$\bar{e}$	0.3384	0.0113	0.0446	0.0811	0.1779	0.1943	0.0080
			σ_e^2	0.0242	0.0325	0.0229	0.0222	0.0184	0.0184	0.0020
			σ_e	0.1556	0.1804	0.1512	0.1489	0.1356	0.1356	0.0451
			MSE	0.1435	0.0327	0.0249	0.0290	0.0513	0.0577	0.0021
17	B-W-R	0.80	$\bar{e}$	-0.4733	-0.0024	0.0083	-0.0807	0.2031	-0.0771	-0.0017
			σ_e^2	0.0560	0.0826	0.0691	0.0688	0.0769	0.0614	0.0014
			σ_e	0.2366	0.2874	0.2629	0.2623	0.2774	0.2477	0.0380
			MSE	0.2894	0.0826	0.0692	0.0756	0.1199	0.0676	0.0014
18	A-W-R	0.80	$\bar{e}$	0.2933	0.0956	0.1646	0.1939	0.4230	0.2122	0.0027
			σ_e^2	0.0600	0.1264	0.0630	0.0563	0.1120	0.0559	0.0006
			σ_e	0.2449	0.3555	0.2511	0.2372	0.3347	0.2365	0.0240
			MSE	0.1496	0.1359	0.1913	0.0954	0.2984	0.1029	0.0006

Continuation of Table 5.14.

Series	Best Comb. of Forecasting Methods	Best Smoothing Constant (α) (For Formula 27)		Average	Formula 10 (ν=6)	Formula 11 (β=6)	Formula 12 (γ=0.80)	Formula 13 (Θ=1.40)	Formula 14 (ϕ=1.20)	Formula 27
19	A-B-W-R	0.4	$\bar{e}$	0.0126	-0.0266	0.0322	0.0407	0.2032	0.0984	-0.0157
			σ_e^2	0.0898	0.4507	0.0902	0.0890	0.1163	0.0890	0.0284
			σ_e	0.2997	0.6713	0.3004	0.2984	0.3411	0.2984	0.1686
			MSE	0.1026	0.4514	0.0913	0.0907	0.1581	0.0988	0.0283
20	B-W-R-S	0.8	$\bar{e}$	-0.2633	0.0832	0.1115	0.0763	0.3111	0.0577	-0.0066
			σ_e^2	2.4138	1.0818	0.3849	0.5039	1.0578	0.3653	0.0027
			σ_e	1.5536	1.0401	0.6204	0.7099	1.0285	0.6044	0.0524
			MSE	2.4855	1.0889	0.3978	0.5099	1.1579	0.3687	0.0028

Table 5.15. Effect of Changing the Smoothing Constant (ρ) When Updating the Weights for Combinations of Two Methods

Series	α	Sum Of Errors	Average Error	Variance Of Forecast Error	Standard Deviation	Mean Square Error
4 W-S	0.0	-4.7885	-0.0319	0.1422	0.3771	0.1432
	0.2	-1.1910	-0.0079	0.0765	0.2767	0.0765
	0.4	-0.8932	-0.0058	0.0414	0.2036	0.0524
	0.6	-0.4778	-0.0032	0.0335	0.1829	0.0335
	0.8	-0.1163	-0.0008	0.0172	0.1313	0.0172
	1.0	-21.2664	-0.2593	10.3353	3.2149	10.4034
9 B-J	0.0	-0.9623	-0.0085	0.0111	0.1055	0.1121
	0.2	-1.2846	-0.0114	0.0077	0.0880	0.0079
	0.4	-1.2087	-0.0107	0.0057	0.0755	0.0582
	0.6	-0.9205	-0.0081	0.0044	0.0664	0.0045
	0.8	-0.7449	-0.0066	0.0032	0.0564	0.0032
	1.0	-0.4241	-0.0038	0.0106	0.1030	0.0162
13 A-W	0.0	-25.3083	-0.3086	10.4606	13.2343	10.5571
	0.2	11.7788	0.1436	8.7949	2.9656	8.8158
	0.4	10.9209	0.1332	6.4764	2.5449	6.4943
	0.6	9.7897	0.1194	3.9189	1.9796	3.9330
	0.8	10.3862	0.1267	2.2944	1.5147	2.3106
	1.0	-21.2664	-0.2593	10.3353	3.2149	10.4034
14 A-R	0.0	-161.1895	-6.4476	10.6440	3.2625	53.9475
	0.2	-36.5395	-1.4616	14.2071	3.7692	16.4323
	0.4	-21.9947	-0.9798	7.5753	2.7523	8.3815
	0.6	-14.4226	-0.5769	3.8863	1.9714	4.2330
	0.8	-8.4023	-0.3361	1.5420	1.2418	1.6596
	1.0	-197.35	-7.8940	15.7980	3.9747	80.7097

Continuation Table 5.15.

Series	α	$\sum e$	$\bar{e}$	V(e)	σ_e	MSE
15 B-W	0.0	4.3030	0.1721	1.9790	1.4068	2.0098
	0.2	12.8813	0.5153	1.4091	1.1871	1.6856
	0.4	16.2782	0.6511	1.3510	1.1623	1.7927
	0.6	16.8480	0.6739	1.3621	1.1671	1.8352
	0.8	14.9808	0.5992	1.3484	1.1612	1.7225
	1.0	-	-	-	-	-
16 W-R	0.0	4.2240	0.1690	0.0184	0.1355	0.0481
	0.2	0.7042	0.0282	0.0163	0.1277	0.0171
	0.4	0.4688	0.0188	0.0109	0.1046	0.0113
	0.6	0.3304	0.0132	0.0060	0.0772	0.0061
	0.8	0.2011	0.0080	0.0020	0.0452	0.0021
	1.0	8.1173	0.3247	0.0289	0.1699	0.1387
17 W-R	0.0	-9.4454	-0.3778	0.1228	0.3505	0.2715
	0.2	-3.8538	-0.1542	0.1319	0.3631	0.1566
	0.4	-2.6857	-0.1074	0.1155	0.3398	0.1275
	0.6	-2.1705	-0.0868	0.0933	0.3055	0.10118
	0.8	-1.7975	-0.0719	0.0715	0.2674	0.0769
	1.0	-18.60	-0.7440	0.0860	0.2933	0.6626
18 W-S	0.0	3.9967	0.1599	0.0629	0.2507	0.08947
	0.2	1.7675	0.0703	0.0465	0.2156	0.0517
	0.4	1.2215	0.0489	0.0303	0.1739	0.0327
	0.6	0.8727	0.0349	0.0167	0.1291	0.0179
	0.8	0.5541	0.0222	0.0056	0.0749	0.0061
	1.0	7.6439	0.3058	0.0650	0.2549	0.1623

Continuation Table 5.15.

Series	α	$\sum e$	$\bar{e}$	V(e)	σ_e	MSE
19 W-S	0.0	1.1779	0.0121	0.0915	0.3025	0.0916
	0.2	-1.2846	-0.0114	0.0077	0.0880	0.0079
	0.4	- .3264	-0.0034	0.0508	0.2254	0.0508
	0.6	- .5421	-0.0056	0.0339	0.1841	0.0339
	0.8	-0.0627	-0.0006	0.0196	0.1399	0.0196
	1.0	0.4128	0.0043	0.0897	0.2995	0.0896
20 A-W	0.0	4.5828	0.1528	0.3392	0.5824	0.3633
	0.2	3.8780	0.1293	0.2235	0.4727	0.2407
	0.4	3.1943	0.1065	0.1451	0.3809	0.1568
	0.6	2.5746	0.0858	0.0896	0.2994	0.0972
	0.8	1.7381	0.0579	0.0479	0.2188	0.0514
	1.0	4.5610	0.1520	0.3468	0.5889	0.3706

Table 5.16. Weight Updating - Effect of a Change in the Smoothing Constant (α) - Combinations of More Than Two Methods

Series	α	$\sum e$	$\bar{e}$	V(e)	σ_e	MSE
4	0.0	0.8804	0.0059	0.3477	0.5896	0.3477
A-W-R-S	0.2	-0.5303	-0.0035	0.0415	0.2037	0.0415
	0.4	0.6155	0.0041	0.0136	0.1166	0.0136
	0.6	0.5564	0.0037	0.0012	0.0344	0.0012
	0.8	-0.1163	-0.0008	0.0172	0.1313	0.0172
	1.0	-	-	-	-	-
9	0.0	1.2463	-0.0110	0.0110	0.1049	0.0111
B-W-S-R	0.2	-0.0235	-0.0002	0.0050	0.0709	0.0050
	0.4	-0.2420	-0.0021	0.0025	0.0505	0.0025
	0.6	-0.3128	-0.0028	0.0011	0.0328	0.0108
	0.8	-0.2413	-0.0021	0.0002	0.0155	0.0002
	1.0	-0.4241	-0.0038	0.0106	0.1030	0.0106
13	0.0	-25.3083	-0.3086	10.4606	3.2343	10.5571
A-W	0.2	11.7788	0.1436	8.7949	2.9656	8.8158
	0.4	10.9209	0.1332	6.4764	2.5949	6.4943
	0.6	9.7897	0.1194	3.9189	1.9796	3.9333
	0.8	10.3862	0.1267	2.2944	1.5147	2.3106
	1.0	-21.2664	-0.2593	10.3353	3.2149	10.4034
14	0.0	-96.7269	-3.8691	8.9006	2.9834	24.4941
A-R-S	0.2	-9.4329	-0.3773	5.2061	2.2817	5.3544
	0.4	-6.3480	-0.2539	1.9912	1.4111	2.0583
	0.6	-4.0108	-0.1604	0.3660	0.6050	0.3928
	0.8	0.5259	0.0210	0.1816	0.4261	0.1820
	1.0	-104.7250	-4.1890	9.2405	3.0398	27.5194

Continuation Table 5.16.

Series	α	$\sum e$	$\bar{e}$	V(e)	σ_e	MSE
15	0.0	20.5466	0.8219	1.9294	1.3890	2.6329
B-W-R-S	0.2	1.7482	0.0699	0.8373	0.9150	0.8424
	0.4	1.1035	0.0441	0.3883	0.6231	0.3903
	0.6	-0.2867	-0.0115	0.3816	0.6177	0.3817
	0.8	-4.6980	-0.1879	1.2594	1.1222	1.2962
	1.0	3.7500	0.1500	2.0629	1.4363	2.0863
16	0.0	4.2240	0.1690	0.0184	0.1355	0.0481
W-R	0.2	0.7042	0.0282	0.0163	0.1277	0.0171
	0.4	0.4688	0.0188	0.0109	0.1046	0.0113
	0.6	0.3304	0.0132	0.0060	0.0772	0.0614
	0.8	0.2009	0.0080	0.0020	0.0451	0.0021
	1.0	8.1173	0.3247	0.0289	0.1699	0.1387
17	0.0	-4.5965	-0.1839	0.0593	0.2436	0.0945
B-W-R	0.2	-1.0110	-0.0404	0.0363	0.1906	0.0380
	0.4	-0.4134	-0.0165	0.0206	0.1434	0.0208
	0.6	0.9026	0.0361	0.4827	0.6948	0.4828
	0.8	-0.0423	-0.0017	0.0014	0.0380	0.0014
	1.0	-11.8333	-0.4733	0.0560	0.2366	0.2893
18	0.0	4.5808	0.1832	0.0562	0.2371	0.0912
A-W-R	0.2	0.9013	0.0361	0.0326	0.1806	0.0339
	0.4	0.3652	0.0146	0.0144	0.1202	0.0147
	0.6	0.1747	0.0070	0.0045	0.0670	0.0454
	0.8	0.0677	0.0027	0.0006	0.0240	0.0006
	1.0	7.3337	0.2933	0.0600	0.2449	0.1496

Continuation Table 5.16.

Series	α	$\sum e$	$\bar{e}$	V(e)	σ_e	MSE
19	0.0	10.5691	0.1090	0.0895	0.2992	0.1015
A-B-W-R-S	0.2	0.7030	0.0072	0.0432	0.2079	0.0432
	0.4	-1.5231	-0.0157	0.0284	0.1686	0.0287
	0.6	-	-	-	-	-
	0.8	-3.3255	-0.0343	0.0621	0.2491	0.0632
	1.0	10.9220	0.1126	0.0898	0.2997	0.1026
20	0.0	2.7373	0.0912	0.3575	0.5978	0.3659
B-W-R-S	0.2	0.7019	0.0234	0.1640	0.4050	0.1646
	0.4	-0.4307	-0.0144	0.0683	0.2613	0.0684
	0.6	-0.5724	-0.0191	0.0202	0.1420	0.0205
	0.8	-0.1990	-0.0066	0.0027	0.0524	0.0028
	1.0	-7.9000	-0.2633	2.4138	1.5536	2.4855

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

In this study the combination of forecasts has been examined. If $\{X_t\}$, t= 1, 2, . . . represents a time series, and $\hat{X}_{i,T+\tau}(T)$ is the forecast of $X_{T+\tau}$ made at origin T by forecasting method i, i=1, 2, . . . , n, then the combined forecast is expressed as

$$\hat{X}_{T+\tau}(T) = \sum_{i=1}^{n} w_{i,T} \hat{X}_{i,T+\tau}(T).$$

Two specific research objectives were accomplished. The first was to find an appropriate procedure for estimating and updating the weights in a system that combines several forecasts to produce a single forecast of a future observation. This method was developed by choosing the weights proportional to the variances of the individual forecast errors. The method minimizes the variance of the combined forecast errors, assuring that the variance will be less than or equal to the individual variances of the methods in the combination. If $\sum_{T+\tau}$ is the covariance matrix, the weights for the combined forecast can be defined as

$$\underline{w}_T = \frac{\sum_{T+\tau}^{-1} \underline{1}}{(\underline{1}' \sum_{T+\tau} \underline{1})}$$

where the elements of the covariance matrix are estimates of the individual variances and correlation coefficients between pairs of methods. The elements of the covariance matrix ($\sum_T$) are expressed as

$$\sum_T = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1n}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2n}^2 \\ \vdots & \vdots & & \vdots \\ \sigma_{n1}^2 & \sigma_{n2}^2 & \cdots & \sigma_{nn}^2 \end{bmatrix} .$$

The second objective was to test the effectiveness of this procedure by using industrial and/or economic time series, with a simulation experiment. An empirical investigation was performed for twenty real time series to test the usefulness of combining forecasts. For each one, models were fitted using five forecasting methods -- Adaptive Filtering, Box-Jenkins, Linear Regression, Exponential Smoothing and Winters' Method. The objectives of the simulation were twofold: to investigate the difference in performance of the five forecasting methods, and to test the usefulness and effectiveness of forecasting and updating forecasts.

The study of the performance of the individual forecasting methods, when applied to the series of data suggest some specific conclusions

1. There is no "one best" forecasting method.
2. The performance of a forecasting method depends exclusively upon the characteristics of the series under study.

3. Box-Jenkins models perform very well when forecasting for short lead times, but its advantages are lost when forecasting over longer lead times.
4. Adaptive Filtering performs well for lead time larger than 1.
5. For a non-stationary time series, a selection of a forecasting method based on minimum mean square error in the modeling phase is not enough evidence to believe that it will also behave well in the forecasting phase.

The study of the usefulness of the combination of forecasts considered three phases: combinations of two forecasting methods, effect of increasing the number of methods, and weight updating. The first phase of the study shows that:

1. Combinations of two methods having a correlation coefficient (between their forecasting errors), close to zero perform well compared to combinations of methods having a high positive correlation coefficient.
2. Combinations involving automatic methods -- (those that do not require the forecaster's intervention) such as Adaptive Filtering, Winters' and Smoothing give results that outperform Box-Jenkins.
3. Combinations of two forecasting methods always behave well considering variance of forecast error. When the number of methods in the combination is increased, the variance of the combined forecast error is decreased, but the mean square error increases.

A comparative study between the developed combination procedure and the heuristic methods suggested by Bates and Granger [14] indicates the advantages of the proposed method. For all the cases studied, the proposed approach outperforms Bates and Granger's techniques by more than 50 percent, in terms of accuracy measured with MSE.

A final objective of the research was to develop and test a method to update the weights used for the combination of forecasts. The procedure suggested was to update the elements of the covariance matrix $\sum$ and use the new matrix for the computation of updated weights. The elements of the matrix $\sum$ are updated using

$$\hat{\sigma}^2_{ii}(t+1) = \alpha\ e^2_{i,t}(t) + (1 - \alpha)\ \hat{\sigma}^2_{ii}(t)$$

and $\sigma^2_{ij}\ (t+1) = \alpha[e_{i,t+1}(t)\ e_{j,t+1}(t)] + (1 - \alpha)\ \hat{\sigma}^2_{ij}\ (t).$

Any value of the smoothing constant (α) results in improvements in the combination procedure, however, it was found that α close to 0.80 gives the best results for the combination.

6.2 Guidelines for the Combination of Forecasts

Several guidelines are recommended to decide when it will be desirable to use the best individual forecast and when to combine it with others(s).

1. Under the assumption of accuracy as the main criteria in the selection of a forecasting method, a combination of forecasts should be selected. It has been shown that the forecasting phase of the combined forecast always outperforms

the forecasting phase of the best individual forecasting method.

2. A combination of two forecasting methods will do well if the correlation coefficient between the forecast errors is close to zero or large and negative.
3. The number of methods to combine depends on the objectives of the analyst. The more methods used in the combination, the less the variance but also the less parsimónius the forecasting model.

6.3 Recommendations

This study has produced several recommendations for further research.

1. Only a point estimate for the future observation is given. The determination of prediction intervals would be useful, based on an optimal combination of forecasts.
2. Combined forecasts for a lead time of one were studied only. Forecasts for other lead times will require the determination of the covariance matrix for lead times greater than one. Montgomery and Johnson [1] suggest an approximation using the forecast error for a lead time of one. This procedure should be investigated for its applicability to the combination of forecasts at lead times greater than 1.
3. A procedure for selecting an optimal smoothing constant (α), to be used in the updating procedure, is required.
4. Since cost is an important factor in the selection of a forecasting technique, it is recommended a feasibility study

for the accuracy gained in the combination versus the cost involved. It is likely that this would have to be done in a specific application, as the cost-benefit relationship on forecasting often depends on the ultimate use of the forecast.

APPENDICES

APPENDIX A

ADAPTIVE FILTERING
Instructions for the Use of ADAP

PURPOSE: This computer program uses Adaptive Filtering method to forecast a time series. It determines the optimal parameters (weights) to be applied to past data in such a way that it minimizes the squared errors.

INPUT PREPARATION:

CARD 1

Columns 1-5	N	Number of data points to be used in identification phase (Integer).
Columns 6-10	N1	Number of data points to be used in forecasting phase (Integer).

CARD 2

Columns 1-8	X(1) X(2) . . .	Vector of observed values dimensioned to a maximum of 500.

Enter the data 8 observations per card in F8.0 FORMAT

CARD 3

Enter a title for the series, restricted to 80 columns

CARD 4

Columns 1-5	NCR	Number of computer runs, where a new computer run means a different learning constant is specified (Integer).

CARD 5

Columns 1-5	NW	Number of weights in the new computer run (Maximum of 25).
Columns 6-10	NI	Number of iterations (Integer)
Columns 11-15	FLAG	There are three options associated with the calculation of weights in this program 0 indicates initial weights are given as part of input

1	initial weights are computed using 1/NW
2	use weights from a previous computer run.

CARD 6

Note: This card required only if flag=0

Columns 1-8	W(1)	
Columns 9-16	W(2)	Enter the weights 8 observations per card (F8.2)
	.	
	.	
	.	
Columns ...	W(NW)	

FLOWCHART:

Input Time Series Values

↓

Starting Values for k and W_i

↓

For each computer run

↓

Begin iterations

↓

For each period

↓

$$\text{Forecast} = \sum W_i X_{t-i+1}$$

$$\text{Error} = X_{t+1} - \text{Forecast}$$

Update Weight:

$$W_i' = W_i + (2)(k)(\text{Error})X_{t-i+1}$$

↓

All periods?

↓

All interations?

↓

All computer runs?

↓

Final weight values

↓

Initialization Phase Forecasts

↓

Forecasting Phase

Listing for Computer Program ADAP

```
      PROGRAM ADAP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DOUBLE PRECISION F,X,W,E,XMSE,SES
      DIMENSION F(500),X(500),TITLE(17),W(25),SW(25),SF(500)
      DIMENSION XOBS(500)
      INTEGER FLAG,H,SCR
      REAL K
C
C
C                FORECASTING WITH ADAPTIVE FILTERING
C
C
C     THE TECHNIQUE OF ADAPTIVE FILTERING STATES HOW THE
C     WEIGHTS SHOULD BE ADJUSTED AFTER THE FORECAST ERROR
C     HAS BEEN COMPUTED.
C
C
C     EQUATION FOR ADJUSTING THE WEIGHTS ONCE THE ERROR HAS
C     BEEN COMPUTED IS
C               W'=W+2KEX
C     WHERE:
C          W'= THE REVISED SET OF WEIGHTS,
C          W = THE OLD SET OF WEIGHTS,
C          K = A CONSTANT TERM REFERRED TO AS THE LEARNING CONSTANT,
C          E = THE ERROR OF THE FORECAST,
C          X = THE OBSERVED VALUES.
C            SES = SUM OF ERRORS SQUARE
C            SEM = STORAGE FOR MIN MEAN SQUARE ERROR
C            SUMW = SUM OF WEIGHTS
C            XMSE = MEAN SQUARE ERROR
C            SCR = STORAGE FOR COMPUTER RUN # WHERE MIN MEAN
C                  SQUARE ERROR OCCURED
C            N  = # OF DATA VALUES TO BE USED IN IDENTIFICATION
C            N1 = # OF DATA VALUES TO BE USED IN FORECASTING
C            N+N1 = TOTAL # OF POINTS IN DATA SET
C
C
      READ(5,2) N,N1
    2 FORMAT(16I5)
C                      INITIATE VARIABLES
      ER=1.E+37
      SEM=9999999.
      SES=0
      N2=N+N1
C                      READ ACTUAL VALUES
      READ(5,3)(X(I),I=1,N2)
    3 FORMAT(8F8.0)
C                    READ TITLE OF THE SERIES
      READ(5,4)(TITLE(J),J=1,16)
      READ(5,4) TITLE(17)
    4 FORMAT(16A5)
      PRINT 1000, (TITLE(J),J=1,16),N2
 1000 FORMAT(1H1,2X,36HFORECASTING WITH ADAPTIVE FILTERING ,/,2X,37('.')
     +,/,2X,16A5,17X,I5,14H  OBSERVATIONS)
      PRINT 1001
 1001 FORMAT(///,2X,13HDATA LISTING:)
      DO 200 I=1,N2,8
      H=MIN0(N2,I+7)
```

```
 200  PRINT 1002,  I,H,(X(J),J=I,H)
 1002 FORMAT(10X,I4,1H-,I4,1X,8E12.6)
      PRINT 1003,(TITLE(J),J=1,16)
 1003 FORMAT(1H1,/////,2X,37HTRAINING THE WEIGHTS FOR FORECASTING ,16A5)
      PRINT 1004
 1004 FORMAT(60X,'NUMBER OF',/,2X,'COMPUTER',5X,'NUMBER OF',5X,'FINAL WE
     +IGHT',5X,'VALUE OF',7X,'TRAINING',7X,'MEAN SQUARE ERROR',/,4X,'RUN'
     + ',8X,'WEIGHTS',8X,'VALUES',12X,'K',10X,'ITERATIONS',6X,'ON FINAL
     +ITERATION',/,2X,92('-'))
      READ(5,2)NCR
      WRITE(6,2) NCR
      DO 300 I=1,NCR
      READ(5,5)NW,NI,K,FLAG
C     WRITE(6,5) NW,NI,K,FLAG
C                     NCR=NUMBER OF COMPUTER RUNS
C                     NW =NUMBER OF WEIGHTS IN THE COMPUTER RUN
C                     NI =NUMBER OF ITERATIONS
C                     FLAG= INDICATES IF INITIAL SET OF WEIGHTS
C                           IS GIVEN OR CALCULATED FROM 1/NW.
C                           FLAG = 0  IMPLIES READ WEIGHTS
C                           FLAG = 1  COMPUTE INITIAL WEIGHTS
C                           FLAG = 2  IMPLIES USE VALUES OF WEIGHTS FROM
C                                     PREVIOUS COMPUTER RUNS
C                      ER = ERROR REDUCTION LIMIT:USED TO STOP
C                           ITERATIONS.
C
  5   FORMAT(2I5,F5.0,I1)
      IF(FLAG.EQ.0) GO TO 51
      IF(FLAG.EQ.2) GO TO 52
      READ(5,6)(W(J),J=1,NW)
  6   FORMAT(13F6.0)
      GO TO 52
  51  DO 301 J=1,NW
 301  W(J)=1./NW
  7   FORMAT(13(F8.2,1X))
C     WRITE(6,7)(W(J),J=1,NW)
C     WRITE(6,9) NI,NW
  9   FORMAT(2(I10))
   52 DO 400 M=1,NI
      NN=N+1
      J=NW
      SES=0.
 302  J=J+1
      F(J)=0.
      IF(J.GE.NN) GO TO 400
      JJJ=J-NW
      H1=0.
      DO 500 JJ=1,NW
      F(J)=W(JJ)*X(JJJ)+F(J)
      H1=H1+X(JJJ)*X(JJJ)
 500  JJJ=JJJ+1
      H1=SQRT(H1)
      E=X(J)-F(J)
      SES=E*E+SES
C                STOPPING RULE FOR NUMBER OF ITERATIONS
      IF(SES+.0001.GT.ER) GO TO 401
      ER=SES
C
 401  JJJ=J-NW
C                                   UPDATE WEIGHTS
      DO 600 JJ=1,NW
      W(JJ)=W(JJ)+2.*K*E/H1*X(JJJ)/H1
      JJJ=JJJ+1
 600  CONTINUE
C     WRITE(6,8) J,X(J),F(J),E,SES,(W(JJ),JJ=1,NW)
  8   FORMAT(I3,4(1X,F8.4),/,12F10.4)
      GO TO 302
```

```
  400 CONTINUE
      NI=M-1
C                 COMPUTE MEAN SQUARE ERROR ON FINAL ITERATION
      XMSE=SES/(N-NW+1)
C      IF ACTUAL MEAN  SQUARE ERROR IS GREATER THAN THE BEST SO FAR
C         GO TO NEXT COMPUTER RUN
      IF(XMSE.GT.SEM) GO TO 303
      SEM=XMSE
C      SAVE WEIGHTS, FORECASTS AND NUMBER OF COMPUTER RUN
      DO 800 MM=1,NW
      SW(MM)=W(MM)
  800 CONTINUE
       SNW=NW
      DO 900 MM=1,N
      SF(MM)=F(MM)
  900 CONTINUE
      SCR=I
  303 DO 991 M=1,NW
      IF(M.EQ.1) PRINT 1005, I,NW,W(M),K,NI,XMSE
      IF(M.NE.1) PRINT 1006,W(M)
  991 CONTINUE
  300 CONTINUE
      PRINT 1007,SCR,SEM
C
C       SUMMARY OF INITIALIZATION STEP
C
      PRINT 1011
      SES=0.
      CE=0.
      MM=SNW+1
      DO 997 I=MM,N
      E=X(I)-SF(I)
      SES=E*E+SES
      CE=CE+E
      VARE=(SES-(CE**2)/N)/(N-1)
      SVARE=SQRT(VARE)
      AE=CE/(N-SNW)
      WRITE(6,1009) I,X(I),SF(I),E,CE,SES
      WRITE(8,1012) I,SF(I),E,X(I),TITLE(17),I
  997 CONTINUE
      WRITE(6,1010) CE,AE,VARE,SVARE,SEM
C
C
C     ***********************************************************
C                          FORECASTING PHASE
C     ***********************************************************
C
C
C     INITIATE VARIABLES:
      IF(N1.EQ.0) STOP
      NW=SNW
       CE=0.
       SES=0
       NN=N+N1+1
      DO 999 I=1,NW
  999 W(I)=SW(I)
       J=N
       WRITE(6,1008)
   995 J=J+1
      IF(J.GE.NN) GO TO 993
      F(J)=0.
       JJJ=J-NW
      H1=0.
       DO 992 JJ=1,NW
       F(J)=W(JJ)*X(JJJ)+F(J)
      H1=H1+X(JJJ)*X(JJJ)
```

```
  992   JJJ=JJJ+1
        E=X(J)-F(J)
        CE=CE+E
        SES=E*E+SES
        JJJ=J-NW
        WRITE(6,1009) J,X(J),F(J),E,CE,SES
C      WRITE(8,1012) J,SF(J),E,X(J),TITLE(17),J
C
C             UPDATE WEIGHTS
C
        DO 994 JJ=1,NW
        W(JJ)=W(JJ)+2.*K*E/H1*X(JJJ)/H1
  994   JJJ=JJJ+1
        GO TO 995
C
C             COMPUTE MEAN SQUARE ERROR FOR FORECASTING PHASE
C
  993   XMSE=SES/(N1-NW+1)
        AE=CE/N1
       VARE=(SES-(CE**2)/N1)/(N1-1)
        SVARE=SQRT(VARE)
        WRITE(6,1010) CE,AE,VARE,SVARE,XMSE
 1008 FORMAT(1H1,30X,'****FORECASTING PHASE****',//,2X,'PERIOD',5X,'OBSER
     +RVATION',5X,'FORECAST',5X,'ERROR',5X,'CUM. ERROR',5X,'SUM OF SQ. ERR
     +RROR',//)
 1009 FORMAT(2X,I6,5X,F10.4,5X,F10.4,F10.4,4X,F10.4,5X,F14.4)
 1010 FORMAT(1H0,2X,'SUM OF FORECAST ERRORS=',F10.4,5X,'AVERAGE FORECAST
     + ERROR=',F10.4,5X,'VARIANCE=',F10.4,5X,'STANDARD DEVIATION='F10.4,
     +/,2X,'MEAN SQUARE ERROR=',E14.6)
 1005 FORMAT(////5X,I2,8X,I2,11X,F8.4,11X,F5.2,12X,I5,8X,F12.4)
 1006 FORMAT(28X,F8.4)
 1007 FORMAT(2X,92(1H-),///,2X,'BEST SET OF WEIGHTS OCCURS AT COMPUTER RUN',I
     +UN',I5,2X,'WHERE MEAN SQUARE ERROR:',E14.6,' IS A MINIMUN')
 1011 FORMAT(1H1,30X,'****INITIALIZATION STEP****',//,2X,'PERIOD',5X,'OBS
     +SERVATION',5X,'FORECAST',5X,'ERROR',5X,'CUM.ERROR',5X,'SUM OF SQ.E
     +RROR',//)
 1012 FORMAT(I3,2X,E14.4,2(2X,E14.4),20X,A6,I3)
      N2=N+N1
      DO 880 J=N,N2
  880 SF(J)=F(J)
      DO 890 J=1,N2
  890 XOBS(J)=X(J)
      CALL PLOT(XOBS,SF,N2,N2,100,1,1,1)
      STOP
      END
      SUBROUTINE PLOT(XOBS,XFOR,NOBS,NFOR,ISCALE,OPTION,JDIV,IDIV)
      DIMENSION XOBS(500),XFOR(1000),GRAPH(100),Y(11)
      DATA OBS,FOR,BLANK,DOT,PLUS,DASH/1HO,1HF,1H ,1H.,1H+,1H-/
C     ******************************************************
C     * XOBS IS A VECTOR OF ACTUAL OBSERVATIONS.           *
C     * XFOR IS THE VECTOR OF FORECASTING.                 *
C     * NOBS IS # OF PERIODS OBSERVED>                     *
C     * NFOR IS # OF PERIODS FORCASTED.                    *
C     * ISCALE SETS THE WIDTH OF THE PLOT. 50 OR 100 COLMNS*
C     * FOR PARTITIONING OF THE GRAPH:                     *
C     *       OPTION=0. WILL SUPPRESS PARTITIONING.        *
C     *       OPTION=1. WILL PROVIDE PARTITIONING,         *
C     *                 EVERY 10TH COL. AND 12TH ROW.      *
C     *       OPTION=2. SET YOUR OWN PARTITIONS.           *
C     *                                                    *
C     * WHEN OPTION 2 IS CHOSEN:                           *
C     *       JDIV IS THE HORIZONTAL PARTITION.            *
C     *       IDIV IS THE VERTICAL PARTITION.              *
C     * WHEN OBSERVED & FORCASTED ARE TO BE PLOTTED        *
C     * TOGETHER, ONLY 'F' WILL BE PRINTED.                *
C     *                                                    *
```

```
C       * THIS PLOTTER WAS CONTRIBUTED TO THE BENEFIT OF       *
C       * ALL BY GADI NAAMAN & ROBERT ALEXANDER.                *
C       *******************************************************
        IF(OPTION.EQ.2.) GO TO 3
        IDIV=12
        JDIV=10
      3 ITOP=NFOR
        RMAX=XOBS(1)
        RMIN=XOBS(1)
        IF(NOBS.GT.NFOR) ITOP=NOBS
        DO 1 I=1,ITOP
        IF(I.GT.NOBS) GO TO 2
        IF(XOBS(I).GE.RMAX) RMAX=XOBS(I)
        IF(XOBS(I).LE.RMIN) RMIN=XOBS(I)
      2 IF(I.GT.NFOR) GO TO 1
        IF(XFOR(I).GE.RMAX) RMAX=XFOR(I)
        IF(XFOR(I).LE.RMIN) RMIN=XFOR(I)
      1 CONTINUE
        SCALE=FLOAT(ISCALE)
        DIV=(RMAX-RMIN)/SCALE
        PRINT*,'RMIN=',RMIN,' RMAX=',RMAX,' OPTION=',OPTION,
       +'JDIV=',JDIV
            ENDFILE 6
        INDX=0
        JSCALE=ISCALE+1
        DO 10 K=1,JSCALE,10
        INDX=INDX+1
        Y(INDX)=RMIN+DIV*(K-1)
     10 CONTINUE
        IF(ISCALE.EQ.50) WRITE(6,15) (Y(I),I=1,INDX)
     15 FORMAT(1H1,15X,*PLOT OF OBSERVED & FORCASTED TIME SERIES*
       +/16X,40(*-*)////1HQ/
       +1X,6(F8.2,2X)/6X,5(*.........+*)/58X,*OBSERV*,3X,*FORCST*/1HR/)
        IF(ISCALE.EQ.100) WRITE(6,20) (Y(I),I=1,INDX)
    20  FORMAT(1H1,45X,'PLOT OF OBSERVED & FORECASTED TIME SERIES'
       +/46X,40('-')////1HQ/
       +1X,11(F8.2,2X)/6X,10(*.........+*)/108X,*OBSERV*,3X,*FORCST*/1HR/)
        DO 8 I=1,ITOP
        IXOBS=DOT
        IXFOR=DOT
        IF(I.LE.NOBS) IXOBS=(XOBS(I)-RMIN)/DIV+.05
        IF(I.LE.NFOR) IXFOR=(XFOR(I)-RMIN)/DIV+.5
        IPART=(I/IDIV)*IDIV
        DO 9 J=1,JSCALE
        JPART=(J/JDIV)*JDIV
        GRAPH(J)=BLANK
        IF(OPTION.EQ.0) GO TO 9
        IF(IPART.EQ.I.AND.((J/2)*2).EQ.J) GRAPH(J)=DASH
        IF(JPART.EQ.J.AND.((I/2)*2).EQ.I) GRAPH(J)=DOT
        IF(IPART.EQ.I.AND.JPART.EQ.J) GRAPH(J)=PLUS
      9 CONTINUE
        IF(IXOBS.NE.DOT.AND.IXOBS.NE.0) GRAPH(IXOBS)=OBS
        IF(IXFOR.NE.DOT.AND.IXFOR.NE.0) GRAPH(IXFOR)=FOR
        ICHAR = I
        IF(I.LE.NOBS.AND.I.LE.NFOR)
       +WRITE(6,17)ICHAR,ISCALE,(GRAPH(J),J=1,ISCALE),XOBS(I),XFOR(I)
        IF(I.GT.NOBS)
       +WRITE(6,17) ICHAR,ISCALE,(GRAPH(J),J=1,ISCALE),XFOR(I),XFOR(I)
        IF(I.GT.NFOR) WRITE(6,17) ICHAR,ISCALE,
       +(GRAPH(J),J=1,ISCALE),XOBS(I)
     17 FORMAT(2X,I3,*+*,=A1,F8.2,*,*,F8.2)
      8 CONTINUE
     16 CONTINUE
        RETURN
        END
```

Example of output for computer program ADAP

FORECASTING WITH ADAPTIVE FILTERING
****
INTERNATIONAL AIRLINE PASSENGERS 144 OBSERVATIONS

DATA LISTING:

1- 8	.112000E+03	.116000E+03	.132000E+03	.129000E+03	.121000E+03	.135000E+03	.148000E+03	.148000E+03
9- 16	.136000E+03	.119000E+03	.104000E+03	.118000E+03	.115000E+03	.126000E+03	.141000E+03	.135000E+03
17- 24	.125000E+03	.149000E+03	.170000E+03	.170000E+03	.158000E+03	.133000E+03	.114000E+03	.140000E+03
25- 32	.145000E+03	.150000E+03	.178000E+03	.163000E+03	.172000E+03	.178000E+03	.199000E+03	.199000E+03
33- 40	.184000E+03	.162000E+03	.146000E+03	.166000E+03	.171000E+03	.180000E+03	.193000E+03	.181000E+03
41- 48	.183000E+03	.218000E+03	.230000E+03	.242000E+03	.209000E+03	.191000E+03	.172000E+03	.194000E+03
49- 56	.196000E+03	.196000E+03	.236000E+03	.235000E+03	.229000E+03	.243000E+03	.264000E+03	.272000E+03
57- 64	.237000E+03	.211000E+03	.180000E+03	.201000E+03	.204000E+03	.188000E+03	.235000E+03	.227000E+03
65- 72	.234000E+03	.264000E+03	.302000E+03	.293000E+03	.259000E+03	.229000E+03	.203000E+03	.229000E+03
73- 80	.242000E+03	.233000E+03	.267000E+03	.269000E+03	.270000E+03	.315000E+03	.364000E+03	.347000E+03
81- 88	.312000E+03	.274000E+03	.237000E+03	.278000E+03	.284000E+03	.277000E+03	.317000E+03	.313000E+03
89- 96	.318000E+03	.374000E+03	.413000E+03	.405000E+03	.355000E+03	.306000E+03	.271000E+03	.306000E+03
97- 104	.315000E+03	.301000E+03	.356000E+03	.348000E+03	.355000E+03	.422000E+03	.465000E+03	.467000E+03
105- 112	.404000E+03	.347000E+03	.305000E+03	.336000E+03	.340000E+03	.318000E+03	.362000E+03	.348000E+03
113- 120	.363000E+03	.435000E+03	.491000E+03	.505000E+03	.404000E+03	.359000E+03	.310000E+03	.337000E+03
121- 128	.360000E+03	.342000E+03	.406000E+03	.396000E+03	.420000E+03	.472000E+03	.548000E+03	.559000E+03
129- 136	.463000E+03	.407000E+03	.362000E+03	.405000E+03	.417000E+03	.391000E+03	.419000E+03	.461000E+03
137- 144	.472000E+03	.535000E+03	.622000E+03	.606000E+03	.508000E+03	.461000E+03	.390000E+03	.432000E+03

Continuation Output for Computer Program ADAP

TRAINING THE WEIGHTS FOR FORECASTING INTERNATIONAL AIRLINE PASSENGERS

5

COMPUTER RUN	NUMBER OF WEIGHTS	FINAL WEIGHT VALUES	VALUE OF K	NUMBER OF TRAINING ITERATIONS	MEAN SQUARE ERROR ON FINAL ITERATION
1	12	.6633 .3056 -.0093 -.0091 -.0790 -.0018 .0351 .0215 -.0358 .0215 -.0070 .2459	.20	80	112.1476
2	12	.7319 .3006 -.0403 .0002 -.0714 .0053 .0265 .0162 -.0345 .0386 -.0186 .2045	.25	80	105.6384
3	12	.7710 .2965 -.0567 .0061 -.0670 .0089 .0200 .0129 -.0324 .0485 -.0229 .1787	.28	80	103.2863
4	12	.7964 .2934 -.0669 .0100 -.0643 .0111 .0153 .0108 -.0302 .0545 -.0249 .1614	.30	80	102.2841
5	12	.8589 .2839 -.0894 .0194 -.0585 .0157 .0033 .0066 -.0223 .0674 -.0281 .1179	.35	80	101.6030

BEST SET OF WEIGHTS OCCURS AT COMPUTER RUN 5 WHERE MEAN SQUARE ERROR= .101603E+03 IS A MINIMU

****INITIALIZATION STEP****

PERIOD	OBSERVATION	FORECAST	ERROR	CUM.ERROR	SUM OF SQ.ERROR
13	115.0000	132.7819	-17.7819	-17.7819	316.1970
14	126.0000	127.5626	-1.5626	-19.3445	318.6388
15	141.0000	140.3160	.6840	-18.6606	319.1066
16	135.0000	135.5312	-.5312	-19.1918	319.3888
17	125.0000	131.2887	-6.2887	-25.4805	358.9362
18	149.0000	142.8473	6.1527	-19.3278	396.7924
19	170.0000	162.4263	7.5737	-11.7541	454.1532
20	170.0000	165.9784	4.0216	-7.7325	470.3266
21	158.0000	156.7854	1.2146	-6.5179	471.8017
22	133.0000	136.3229	-3.3229	-9.8408	482.8435
23	114.0000	121.4858	-7.4858	-17.3266	538.8801
24	140.0000	124.5677	15.4323	-1.8943	777.0348
25	145.0000	137.6541	7.3459	5.4516	830.9970
26	150.0000	154.8350	-4.8350	.6166	854.3739
27	178.0000	165.0780	12.9220	13.5386	1021.3516
28	163.0000	167.0968	-4.0968	9.4417	1038.1357
29	172.0000	158.8624	13.1376	22.5794	1210.7331
30	178.0000	199.3705	-21.3705	1.2089	1667.4311
31	199.0000	203.4151	-4.4151	-3.2062	1686.9239
32	199.0000	200.8144	-1.8144	-5.0205	1690.2158
33	184.0000	183.6933	.3067	-4.7139	1690.3099
34	162.0000	154.4638	7.5362	2.8223	1747.1040
35	146.0000	146.0584	-.0584	2.7639	1747.1074
36	166.0000	166.6383	-2.6383	.1256	1754.0679
37	171.0000	171.0209	-.0209	.1048	1754.0684
38	180.0000	184.1456	-4.1456	-4.0409	1771.2546
39	193.0000	201.4463	-8.4463	-12.4872	1842.5945
40	181.0000	185.8078	-4.8078	-17.2949	1865.7091
41	183.0000	189.2157	-6.2157	-23.5106	1904.3441
42	218.0000	198.0012	19.9988	-3.5118	2304.2976
43	230.0000	235.1665	-5.1665	-8.6783	2330.9903
44	242.0000	228.7636	13.2364	4.5581	2506.1917
45	209.0000	223.9915	-14.9915	-10.4335	2730.9379
46	191.0000	183.7637	7.2363	-3.1971	2783.3026
47	172.0000	179.1177	-7.1177	-10.3148	2833.9637
48	194.0000	188.7143	5.2857	-5.0291	2861.9019
49	196.0000	200.9816	-4.9816	-10.0107	2886.7177
50	196.0000	206.6336	-10.6336	-20.6443	2999.7915
51	236.0000	208.3728	27.6272	6.9829	3763.0553
52	235.0000	218.1227	16.8773	23.8603	4047.8999
53	229.0000	241.5543	-12.5543	11.3059	4205.5113
54	243.0000	267.4133	-24.4133	-13.1074	4801.5200
55	264.0000	268.8405	-4.8405	-17.9478	4824.9502
56	272.0000	268.2174	3.7826	-14.1653	4839.2581
57	237.0000	240.9799	-3.9799	-18.1451	4855.0975
58	211.0000	212.7155	-1.7155	-19.8607	4858.0406
59	180.0000	196.9758	-16.9758	-36.8365	5146.2180
60	201.0000	199.9633	1.0367	-35.7998	5147.2928
61	204.0000	201.9050	2.0950	-33.7048	5151.6818
62	188.0000	212.3512	-24.3512	-58.0559	5744.6622
63	235.0000	229.1297	5.8703	-52.1857	5779.1222
64	227.0000	234.2628	-7.2628	-59.4485	5831.8705
65	234.0000	224.6376	9.3624	-50.0861	5919.5252
66	264.0000	253.2698	10.7302	-39.3559	6034.6615
67	302.0000	287.1500	14.8500	-24.5059	6255.1831
68	293.0000	299.2596	-6.2596	-30.7655	6294.3656
69	259.0000	259.7455	-.7455	-31.5110	6294.9214
70	229.0000	225.8001	3.1999	-28.3112	6305.1605
71	203.0000	199.8939	3.1061	-25.2051	6314.8081
72	229.0000	219.6445	9.3555	-15.8495	6402.3343
73	242.0000	222.3514	19.6486	3.7991	6788.4033
74	233.0000	234.9089	-1.9089	1.8902	6792.0471
75	267.0000	269.8056	-2.8056	-.9154	6799.9184
76	269.0000	265.2581	3.7419	2.8265	6813.9201
77	270.0000	278.7885	-8.7885	-5.9620	6891.1570
78	315.0000	313.2517	1.7483	-4.2137	6894.2136
79	364.0000	353.4147	10.5853	6.3716	7006.2619
80	347.0000	349.0844	-2.0844	4.2872	7010.6065

SUM OF FORECAST ERRORS= 4.2872 AVERAGE FORECAST ERROR= .0630 VARIANCE= 88.7389 STANDARD DEVIATION= 9.4201

MEAN SQUARE ERROR= .101603E+03

1

Continuation Output for Computer Program ADAP

****FORECASTING PHASE****

PERIOD	OBSERVATION	FORECAST	ERROR	CUM. ERROR	SUM OF SQ. ERROR
81	312.0000	311.6143	.3857	.3857	.1487
82	274.0000	276.1448	-2.1448	-1.7591	4.7488
83	237.0000	252.4936	-15.4936	-17.2527	244.7997
84	278.0000	275.0549	2.9451	-14.3076	253.4732
85	284.0000	285.4455	-1.4455	-15.7531	255.5628
86	277.0000	282.9775	-5.9775	-21.7306	291.2933
87	317.0000	312.5957	4.4043	-17.3263	310.6915
88	313.0000	315.9978	-2.9978	-20.3241	319.6786
89	318.0000	324.3210	-6.3210	-26.6451	359.6336
90	374.0000	383.0545	-9.0545	-35.6997	441.6182
91	413.0000	430.7441	-17.7441	-53.4438	756.4721
92	405.0000	410.2895	-5.2895	-58.7333	784.4512
93	355.0000	374.8377	-19.8377	-78.5711	1177.9871
94	306.0000	324.9480	-18.9480	-97.5190	1537.0132
95	271.0000	296.3775	-25.3775	-122.8965	2181.0303
96	306.0000	329.4759	-23.4759	-146.3724	2732.1483
97	315.0000	332.5714	-17.5714	-163.9438	3040.9008
98	301.0000	334.1688	-33.1688	-197.1126	4141.0688
99	356.0000	366.3321	-10.3321	-207.4446	4247.8207
100	348.0000	366.0614	-18.0614	-225.5060	4574.0335
101	355.0000	381.2462	-26.2462	-251.7522	5262.8953
102	422.0000	447.6740	-25.6740	-277.4262	5922.0526
103	465.0000	490.8547	-25.8547	-303.2810	6590.5192
104	467.0000	475.7461	-8.7461	-312.0270	6667.0129
105	404.0000	425.5064	-21.5064	-333.5334	7129.5383
106	347.0000	366.1963	-19.1963	-352.7298	7498.0368
107	305.0000	336.3817	-31.3817	-384.1114	8482.8471
108	336.0000	364.7283	-28.7283	-412.8397	9308.1607
109	340.0000	366.6040	-26.6040	-439.4437	10015.9341
110	318.0000	365.4465	-47.4465	-486.8902	12267.1008
111	362.0000	408.2902	-46.2902	-533.1804	14409.8825
112	348.0000	402.0679	-54.0679	-587.2483	17333.2197
113	363.0000	421.6868	-58.6868	-645.9351	20777.3624
114	435.0000	498.4032	-63.4032	-709.3382	24797.3227
115	491.0000	549.3442	-58.3442	-767.6824	28201.3649
116	505.0000	541.8249	-36.8249	-804.5073	29557.4401
117	404.0000	480.0057	-76.0057	-880.5131	35334.3134
118	359.0000	408.3797	-49.3797	-929.8928	37772.6722
119	310.0000	375.3255	-65.3255	-995.2183	42040.0958
120	337.0000	395.9099	-58.9099	-1054.1282	45510.4716
121	360.0000	393.2062	-33.2062	-1087.3344	46613.1201
122	342.0000	384.0647	-42.0647	-1129.3991	48382.5578
123	406.0000	414.3557	-8.3557	-1137.7548	48452.3762
124	396.0000	407.3707	-11.3707	-1149.1255	48581.6689
125	420.0000	436.4597	-16.4597	-1165.5853	48852.5920
126	472.0000	520.8579	-48.8579	-1214.4431	51239.6825
127	548.0000	587.6367	-39.6367	-1254.0798	52810.7516
128	559.0000	582.5331	-23.5331	-1277.6129	53364.5572
129	463.0000	488.6440	-25.6440	-1303.2569	54022.1698
130	407.0000	428.6741	-21.6741	-1324.9310	54491.9380
131	362.0000	383.5623	-21.5623	-1346.4933	54956.8690
132	405.0000	407.2987	-2.2987	-1348.7920	54962.1532
133	417.0000	420.4493	-3.4493	-1352.2413	54974.0509
134	391.0000	420.5108	-29.5108	-1381.7521	55844.9367
135	419.0000	466.9888	-47.9888	-1429.7409	58147.8633
136	461.0000	463.2926	-2.2926	-1432.0335	58153.1195
137	472.0000	499.0830	-27.0830	-1459.1165	58886.6087
138	535.0000	567.8067	-32.8067	-1491.9232	59962.8872
139	622.0000	656.1335	-34.1335	-1526.0567	61127.9797
140	606.0000	648.5134	-42.5134	-1568.5701	62935.3720
141	508.0000	554.0266	-46.0266	-1614.5967	65053.8174
142	461.0000	485.1169	-24.1169	-1638.7135	65635.4402
143	390.0000	450.8130	-60.8130	-1699.5266	69333.6670
144	432.0000	479.1863	-47.1863	-1746.7129	71560.2182

SUM OF FORECAST ERRORS=-1746.7129 AVERAGE FORECAST ERROR= -27.2924 VARIANCE= 379.1785 STANDARD DEVIATION= 19.4725

MEAN SQUARE ERROR= .135019E+04

RMIN=0. RMAX=656.13345032 OPTION=0. JDIV=10

APPENDIX B

PURPOSE: The following programs were written for the forecasting step for the lead times greater than 1. Various specific programs were written for each series.

INPUT PREPARATION:

CARD 1

Columns 1-5	NOB	Number of observations (Total) FORMAT-I5
Columns 6-10	N1	Number of observations used in modeling
Columns 11-15	INDEX	Identifies the Series number
Columns 16-20	NP	Number of parameters in the forecasting model -I5
Columns 21-25	LT	Lead time -I5
Columns 26-30	P(1)	Estimated value for each parameter, read with FORMAT 8.6
	P(2)	
	.	
	.	
	.	
	P(NP)	
Columns . . .	MEAN	Average value for the series

CARD 2

Enter a title for the series, restricted to 80 columns

CARD 3

Enter the data 8 observations per card in F8.0 FORMAT

Box-Jenkins- Multiplicative Model- (0,1,1)x(0,1,1)12- Series 1
Lead Time > 1

```
PROGRAM HELP        73/74    OPT=1                    FTN 4.6+460        78/04/25. 19.11.13

      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C ***********************************************************************
C
C         SERIES 1
C
C ***********************************************************************
      DO 6040 I=1,13
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=14
      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=ALOG(X(I-1))+ALOG(X(I-12))-ALOG(X(I-13))-(P(1)*A(I-1))-(P(2)*
     +A(I-12))+(P(1)*P(2)*A(I-13))
      A(I)=ALOG(X(I))-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
 6000 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      SA=0.
      SAS=0.
      WRITE(6,2060)
      DO 6010 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6010 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
```

Continuation Box-Jenkins - Series 1- Lead Time > 1


```
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=14,N1)
      DO 6022 I=14,N1
      F(I)=ALOG(F(I))
 6022 FI(I)=F(I)
      NN=N1+1
      DO 6020 II=NN,K
      N=II-LT+1
      DO 6021 I=N,II
      IF(I.EQ.N) GO TO 6023
      IF(I.NE.N) GO TO 6024
 6023 Z1=ALOG(X(I-1))
      A1=Z1-FI(I-1)
      GO TO 6025
 6024 Z1=FI(I-1)
      A1=0.
 6025 IF((LT-13).LE.0.OR.I-13.LT.N) A3=ALOG(X(I-13))-F(I-13)
      IF((LT-13).LE.0.OR.I-13.LT.N) Z3=ALOG(X(I-13))
      IF((LT-13).GT.0.AND.I-13.GE.N)A3=0.
      IF(LT-13.GT.0.AND.I-13.GE.N) Z3=F(I-13)
      IF((LT-12).LE.0.OR.I-12.LT.N) A2=ALOG(X(I-12))-F(I-12)
      IF((LT-12).LE.0.OR.I-12.LT.N) Z2=ALOG(X(I-12))
      IF((LT-12).GT.0.AND.I-12.GE.N) Z2=FI(I-12)
      IF((LT-12).GT.0.AND.I-12.GE.N) A2=0.
      FI(I)=Z1+Z2-Z3-(P(1)*A1)+(P(1)*P(2)*A3)
 6021 CONTINUE
      I=II
      F(I)=FI(I)
      A(I)=ALOG(X(I))-F(I)
      IF(II.GE.N1+1) SA=A(I)+SA
      IF(II.GE.N1+1) SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      IF(I.GE.N1+1) WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
      IF(I.GE.N1+1) WRITE(8,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      WRITE(6,2060)
      J=N1+1
      SA=0.
      SAS=0.
      DO 6030 I=J,K
```

Continuation Box-Jenkins - Series 1- Lead Time > 1

```
OGRAM HELP          73/74    OPT=1                         FTN 4.6+460          78/04/25. 19.11.13

            FL=EXP(F(I))
            AL=X(I)-FL
            SA=AL+SA
            SAS=AL**2+SAS
            WRITE(6,2040) I,X(I),FL,AL,SA,SAS
            WRITE(8,2070) I,FL,AL,X(I)
      6030  CONTINUE
            XMSE=SAS/(K-J-1)
            VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
            STD=SQRT(VAR)
            AVE=SA/(K-J)
            WRITE(6,2224)
            WRITE(6,2050) SA,AVE,VAR,STD,XMSE
C                ***** FORMATS *****
      1000  FORMAT(5I5,6F8.6,F14.6)
      1010  FORMAT(8F6.0)
      1020  FORMAT(16A5)
      1030  FORMAT(5X,E14.4,2X,F14.4)
      2000  FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
      2010  FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
           +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
           +,10X,"REGULAR MOVING AVERAGE PARAMETER",F8.6,/,10X,"SEASONAL MOVING AVERAG
           +G AVERAGE PARAMETER",F8.6)
      2020  FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
           +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
           + ERROR",//)
      2030  FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
           +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
           +ERROR",//)
      2040  FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
      2050  FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
           +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
           +0X,"MEAN SQUARE ERROR ",F10.4)
      2060  FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
           +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
           + ERROR",//)
      2070  FORMAT(I3,2X,E14.4,2(2X,E14.4))
      2222  FORMAT(1H1)
      2223  FORMAT(////)
       999  STOP
            END
```

Box-Jenkins- IMA(1,1)- Series 2,3,4,17- Lead Time > 1

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(6),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C*********************************************************************
C
C         SERIES  2
C
C*********************************************************************
      DO 6040 I=1,14
      A(I)=0.
6040  F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=2
      K=N1
      L=0
7010  IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=X(I-1)-(P(1)*A(I-1))
      A(I)=X(I)-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(8,2070) I,F(I),A(I),X(I)
6000  CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
7020  WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=2,N1)
      DO 6022 I=2,N1
6022  FI(I)=F(I)
      NN=N1+1
      DO 6020 IT=NN,K
      N=IT-LT+1
      DO 6021 I=N,IT
      IF(I.EQ.N) GO TO 6023
      IF(I.NE.N) GO TO 6024
6023  Z1=X(I-1)
      A1=Z1-F(I-1)
      GO TO 6025
6024  Z1=FI(I-1)
```

Continuation Box-Jenkins Series 2- Lead Time > 1

```
      A1=0.
6025  F1(I)=Z1-(P(1)*A1)
6021  CONTINUE
      I=IT
      F(I)=F1(I)
      A(I)=X(I)-F(I)
      IF(I1.GE.N1+1) SA=A(I)+SA
      IF(I1.GE.N1+1) SAS=A(I)**2+SAS
      IF(I.GE.N1+1) WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(6,2070) I,F(I),A(I),X(I)
      IF(I.GE.N1+1) WRITE(6,2070) I,F(I),A(I),X(I)
6020  CONTINUE
      XMSE=SAS/(K-J)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
C         ***** FORMATS *****
1000  FORMAT(5I5,8F8.6,F14.6)
1010  FORMAT(8F8.0)
1020  FORMAT(16A5)
1030  FORMAT(5X,E14.4,2X,F14.4)
2000  FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
2010  FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
     +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
     +,10X,"MOVING AVERAGE PARAMETER",F8.6)
2020  FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
     +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
     + ERROR",//)
2030  FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
     +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
     +ERROR",//)
2040  FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
2050  FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
     +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
     +0X,"MEAN SQUARE ERROR ",F10.4)
2060  FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
     +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
     + ERROR",//)
2070  FORMAT(I5,2X,F14.4,2(2X,E14.4))
2222  FORMAT(1H1)
2223  FORMAT(////)
9999  STOP
      END
```

Box-Jenkins-ARIMA(1,1,0)- Series 6- Lead Time > 1

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C ***********************************************************************
C
C        SERIES  6
C
C ***********************************************************************
      DO 6040 I=1,14
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=3
      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=((1.+P(1))*X(I-1))-(P(1)*X(I-2))
      A(I)=X(I)-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(8,2070) I,F(I),A(I),X(I)
 6000 CONTINUE
      XMSE=SAS/72.
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=3,N1)
      DO 6022 I=3,N1
 6022 FI(I)=F(I)
      NN=N1+1
      DO 6020 II=NN,K
      N=II-LT+1
      DO 6021 I=N,II
      IF(I.EQ.N) Z1=X(I-1)
      IF(I.NE.N) Z1=FI(I-1)
      IF((LT-2).LE.0.OR.I-2.LT.N) Z2=X(I-2)
      IF(LT-2.GT.0.AND.I-2.GE.N) Z2=FI(I-2)
      FI(I)=((1.+P(1))*Z1)-(P(1)*Z2)
 6021 CONTINUE
      I=II
```

Continuation- Box-Jenkins Seires 6- Lead Time > 1

```
      F(I)=FI(I)
      A(I)=X(I)-F(I)
      IF(II.GE.N1+1) SA=A(I)+SA
      IF(II.GE.N1+1) SAS=A(I)**2+SAS
      IF(I.GE.N1+1) WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(8,2070) I,F(I),A(I),X(I)
      IF(I.GE.N1+1) WRITE(8,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
C          ***** FORMATS *****
 1000 FORMAT(5I5,8F8.6,F14.6)
 1010 FORMAT(8F8.0)
 1020 FORMAT(16A5)
 1030 FORMAT(5X,E14.4,2X,E14.4)
 2000 FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
 2010 FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
     +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
     +,10X,"REGULAR AUTOREGRESSIVE PARAMETER",F8.6)
 2020 FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
     +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
     + ERROR",//)
 2030 FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
     +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
     +ERROR",//)
 2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
 2050 FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
     +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
     +0X,"MEAN SQUARE ERROR ",F10.4)
 2060 FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
     +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
     + ERROR",//)
 2070 FORMAT(I3,2X,E14.4,2(2X,E14.4))
 2222 FORMAT(1H1)
 2223 FORMAT(////)
  999 STOP
      END
```

Box-Jenkins- AR(3)- Series 7- Lead Time > 1

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(6),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,NI,INDEX,NP,LI,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,NI,LI,NP,(P(I),I=1,NP),MEAN
***********************************************************************

          SERIES  7

***********************************************************************
      DO 6040 I=1,14
      A(I)=0.
 6040 F(I)=0.
      IF(LI.NE.1) GO TO 7020
      J=4
      K=NI
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      X1=X(I-1)-MEAN
      X2=X(I-2)-MEAN
      X3=X(I-3)-MEAN
      F(I)=(P(1)*X1)+(P(2)*X2)+(P(3)*X3)+MEAN
      A(I)=X(I)-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(6,2070) I,F(I),A(I),X(I)
 6000 CONTINUE
      XMSE=SAS/(K-J-3)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=NI+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=NI+2-LI
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=3,NI)
      DO 6022 I=3,NI
 6022 FI(I)=F(I)
      NN=NI+1
      DO 6020 IT=NN,K
      N=IT-LI+1
      DO 6021 I=N,IT
      IF(I.EQ.N) Z1=X(I-1)
      IF(I.NE.N) Z1=FI(I-1)
      IF(LI-2.LE.0.OR.I-2.LT.N) Z2=X(I-2)
```

Continuation - Box-Jenkins- Series 7- Lead Time > 1

```
      IF(I-2.GT.0.AND.I-2.GE.N) Z2=FT(I-2)
      IF(I-3.LE.0.OR.I-3.LT.N) Z3=X(I-3)
      IF(I-3.GT.0.AND.I-3.GE.N) Z3=FT(I-3)
      X1=Z1-MEAN
      X2=Z2-MEAN
      X3=Z3-MEAN
      FT(I)=(P(1)*X1)+(P(2)*X2)+(P(3)*X3)+MEAN
 6021 CONTINUE
      I=IT
      F(I)=FT(I)
      A(I)=X(I)-F(I)
      IF(II.GE.N1+1) SA=A(I)+SA
      IF(II.GE.N1+1) SAS=A(I)**2+SAS
      IF(I.GE.N1+1) WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(6,2070) I,F(I),A(I),X(I)
      IF(I.GE.N1+1) WRITE(6,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J-3)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
          ***** FORMATS *****
 1000 FORMAT(5I5,8F8.6,F14.6)
 1010 FORMAT(8F8.0)
 1020 FORMAT(16A5)
 1030 FORMAT(5X,E14.4,2X,E14.4)
 2000 FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
 2010 FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
     +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
     +,10X,"REGULAR AUTOREGRESSIVE PARAMETERS",F9.6,/,43X,F9.6/,44X,F9.6
     +,/,T11,"MEAN",T44,F9.6)
 2020 FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
     +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
     + ERROR",//)
 2030 FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
     +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
     +ERROR",//)
 2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
 2050 FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
     +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
     +0X,"MEAN SQUARE ERROR ",F10.4)
 2060 FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
     +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
     + ERROR",//)
 2070 FORMAT(I5,2X,E14.4,2(2X,E14.4))
 2222 FORMAT(1H1)
 2223 FORMAT(////)
 9999 STOP
      END
```

Box- Jenkins- ARIMA (1,1,0)- Series 9- Lead Time > 1

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C *********************************************************************
C
C         SERIES  9
C
C *********************************************************************
      DO 6040 I=1,14
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=3
      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=((1.+P(1))*X(I-1))-(P(1)*X(I-2))
      A(I)=X(I)-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(8,2070) I,F(I),A(I),X(I)
 6000 CONTINUE
      XMSE=SAS/110.
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=3,N1)
      DO 6022 I=3,N1
 6022 FI(I)=F(I)
      NN=N1+1
      DO 6020 II=NN,K
      N=II-LT+1
      DO 6021 I=N,II
      IF(I.EQ.N) Z1=X(I-1)
      IF(I.NE.N) Z1=FI(I-1)
      IF((LT-2).LE.0.OR.I-2.LT.N) Z2=X(I-2)
      IF(LT-2.GT.0.AND.I-2.GE.N) Z2=FI(I-2)
      FI(I)=((1.+P(1))*Z1)-(P(1)*Z2)
 6021 CONTINUE
      I=II
```

Continuation - Box_Jenkins- Series 9- Lead Time > 1

```
      F(I)=FI(I)
      A(I)=X(I)-F(I)
      IF(II.GE.N1+1)  SA=A(I)+SA
      IF(II.GE.N1+1)  SAS=A(I)**2+SAS
      IF(I.GE.N1+1)  WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(8,2070)  I,F(I),A(I),X(I)
      IF(I.GE.N1+1)  WRITE(8,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050)  SA,AVE,VAR,STD,XMSE
C         ***** FORMATS *****
 1000 FORMAT(5I5,8F8.6,F14.6)
 1010 FORMAT(8F8.0)
 1020 FORMAT(16A5)
 1030 FORMAT(5X,E14.4,2X,E14.4)
 2000 FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
 2010 FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
     +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
     +,10X,"REGULAR AUTOREGRESSIVE PARAMETER",F8.6)
 2020 FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
     +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
     + ERROR",//)
 2030 FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
     +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
     +ERROR",//)
 2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
 2050 FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
     +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
     +0X,"MEAN SQUARE ERROR ",F10.4)
 2060 FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
     +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
     + ERROR",//)
 2070 FORMAT(I3,2X,E14.4,2(2X,E14.4))
 2222 FORMAT(1H1)
 2223 FORMAT(////)
  999 STOP
      END
```

Box-Jenkins- ARIMA (1,2,0) Log- Series 12- Lead Time > 1

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C ********************************************************************
C
C          SERIES 12
C
C ********************************************************************
      DO 6040 I=1,13
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=14
      K=N1
      L=0
 7010 IF(J.LT.66) WRITE(6,2020)
      IF(J.GE.66) WRITE(6,2030)
      IF(J.GE.66) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=ALOG(X(I-1))+ALOG(X(I-12))-ALOG(X(I-13))-(P(1)*A(I-1))-(P(2)*
     +A(I-12))+(P(1)*P(2)*A(I-13))
      A(I)=ALOG(X(I))-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
 6000 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      SA=0.
      SAS=0.
      WRITE(6,2060)
      DO 6010 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6010 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
```

Continuation Box-Jenkins- Series 12- Lead Time > 1

```
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=14,N1)
      DO 6022 I=14,N1
      F(I)=ALOG(F(I))
 6022 FI(I)=F(I)
      NN=N1+1
      DO 6020 II=NN,K
      N=II-LT+1
      DO 6021 I=N,II
      IF(I.EQ.N) GO TO 6023
      IF(I.NE.N) GO TO 6024
 6023 Z1=ALOG(X(I-1))
      A1=Z1-FI(I-1)
      GO TO 6025
 6024 Z1=FI(I-1)
      A1=0.
 6025 IF((LT-2).LE.0.OR.I-2.LT.N) A2=ALOG(X(I-2))-F(I-2)
      IF(LT-2.GT.0.AND.I-2.GE.N) A2=0.
      IF((LT-13).LE.0.OR.I-13.LT.N) A3=ALOG(X(I-13))-F(I-13)
      IF((LT-13).LE.0.OR.I-13.LT.N) Z3=ALOG(X(I-13))
      IF((LT-13).GT.0.AND.I-13.GE.N)A3=0.
      IF(LT-13.GT.0.AND.I-13.GE.N) Z3=FI(I-13)
      IF((LT-12).LE.0.OR.I-12.LT.N) A2=ALOG(X(I-12))-F(I-12)
      IF((LT-12).LE.0.OR.I-12.LT.N) Z2=ALOG(X(I-12))
      IF((LT-12).GT.0.AND.I-12.GE.N) Z2=FI(I-12)
      IF((LT-12).GT.0.AND.I-12.GE.N) A2=0.
      FI(I)=Z1+Z2-Z3-(P(1)*A1)+(P(1)*P(2)*A3)-(P(2)*A2)
 6021 CONTINUE
      I=II
      F(I)=FI(I)
      A(I)=ALOG(X(I))-F(I)
      IF(II.GE.N1+1) SA=A(I)+SA
      IF(II.GE.N1+1) SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      IF(I.GE.N1+1) WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
      IF(I.GE.N1+1) WRITE(8,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      WRITE(6,2060)
      J=N1+1
      SA=0.
      SAS=0.
      DO 6030 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6030 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
```

Continuation Box-Jenkins- Series 12- Lead Time > 1

```
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
C        ***** FORMATS *****
 1000 FORMAT(5I5,8F8.6,F14.6)
 1010 FORMAT(8F8.0)
 1020 FORMAT(16A5)
 1030 FORMAT(5X,E14.4,2X,E14.4)
 2000 FORMAT(2X,'SERIES',I2,1X,16A5,/,2X,16A5)
 2010 FORMAT(5X,'NUMBER OF OBSERVATIONS',I5,/,5X,'OBSERVATIONS USED ON M
     +ODELING ',I5,/,5X,'LEAD TIME ',I5,/,5X,'NUMBER OF PARAMETERS',I5,/
     +,10X,'REGULAR MOVING AVERAGE PARAMETER',F8.6,/,10X,'SEASONAL MOVING
     +G AVERAGE PARAMETER',F8.6)
 2020 FORMAT(//,30X,'**** INITIALIZATION PHASE ****',//,2X,'PERIOD',5X,'OB
     +OBSERVATION',5X,'FORECAST',5X,'ERROR',5X,'CUM.ERROR',5X,'SUM OF SQ.
     + ERROR',//)
 2030 FORMAT(//,30X,'**** FORECASTING PHASE ****',//,2X,'PERIOD',5X,'OBSE
     +ERVATION',5X,'FORECAST',5X,'ERROR',5X,'CUM. ERROR',5X,'SUM OF SQ. ER
     +ERROR',//)
 2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
 2050 FORMAT(10X,'SUM OF ERROR ',F12.4,/,10X,'AVERAGE ERROR ',F10.4,/,10
     +X,'VARIANCE OF ERROR ',F10.4,/,10X,'STANDARD DEVIATION ',F10.4,/,1
     +0X,'MEAN SQUARE ERROR ',F10.4)
 2060 FORMAT(1H1,30X,'**** ANTILOGS OF ABOVE ****',//,2X,'PERIOD',5X,'OB
     +SERVATION',5X,'FORECAST',5X,'ERROR',5X,'CUM. ERROR',5X,'SUM OF SQ.
     + ERROR',//)
 2070 FORMAT(I3,2X,E14.4,2(2X,E14.4))
 2222 FORMAT(1H1)
 2223 FORMAT(////)
  999 STOP
      END
SCAN  157 EOR    157
```

Box-Jenkins- ARIMA (1,2,0) log- Series 13- Lead Time > 1

PROGRAM HELP 74/74 OPT=1 FTN 4.6+460 78/04/22. 15.19.37

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),F1(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C **********************************************************************
C
C          SERIES 13
C
C **********************************************************************
      DO 6040 I=1,14
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=4
      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=(2+P(1))*ALOG(X(I-1))-(2*P(1)+1)*ALOG(X(I-2))+P(1)*ALOG(X(I-3
     +))
      A(I)=ALOG(X(I))-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
 6000 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      SA=0.
      SAS=0.
      WRITE(6,2060)
      DO 6010 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(6,2070) I,FL,AL,X(I)
 6010 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
```

Continuation Box-Jenkins - Series 13- Lead Time > 1


```
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=4,N1)
      DO 6022 I=4,N1
      F(I)=ALOG(F(I))
 6022 FI(I)=F(I)
      NN=N1+1
      DO 6020 II=NN,K
      N=II-LT+1
      DO 6021 I=N,II
      IF(I.EQ.N) GO TO 6023
      IF(I.NE.N) GO TO 6024
 6023 Z1=ALOG(X(I-1))
      A1=Z1-F(I-1)
      GO TO 6025
 6024 Z1=FI(I-1)
      A1=0.
 6025 IF((LT-2).LE.0.OR.I-2.LT.N) A2=ALOG(X(I-2))-F(I-2)
      IF((LT-2).GT.0.AND.I-2.GE.N) A2=0.
      IF((LT-3).LE.0.OR.I-3.LT.N) A3=ALOG(X(I-3))-F(I-3)
      IF((LT-3).LE.0.OR.I-3.LT.N) Z3=ALOG(X(I-3))
      IF((LT-3).GT.0.AND.I-3.GE.N) Z3=FI(I-3)
      IF((LT-3).GT.0.AND.I-3.GE.N) A3=0.
      FI(I)=(2+P(1))*Z1-(1+(2*P(1)))*Z2+P(1)*Z3
 6021 CONTINUE
      I=II
      F(I)=FI(I)
      A(I)=ALOG(X(I))-F(I)
      IF(II.GE.N1+1) SA=A(I)+SA
      IF(II.GE.N1+1) SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      IF(I.GE.N1+1) WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
      IF(I.GE.N1+1) WRITE(8,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      WRITE(6,2060)
      J=N1+1
      SA=0.
      SAS=0.
      DO 6030 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
```

Continuation Box-Jenkins - Series 13-Lead Time > 1


```
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6030 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
C          ***** FORMATS *****
 1000 FORMAT(5I5,8F8.6,F14.6)
 1010 FORMAT(8F8.0)
 1020 FORMAT(16A5)
 1030 FORMAT(5X,E14.4,2X,F14.4)
 2000 FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
 2010 FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
     +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
     +,10X,"REGULAR AUTOREGRESSIVE PARAMETER",F8.6)
 2020 FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
     +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
     + ERROR",//)
 2030 FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
     +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
     +ERROR",//)
 2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
 2050 FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
     +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
     +0X,"MEAN SQUARE ERROR ",F10.4)
 2060 FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
     +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
     + ERROR",//)
 2070 FORMAT(I3,2X,E14.4,2(2X,E14.4))
 2222 FORMAT(1H1)
 2223 FORMAT(////)
  999 STOP
      END
```

Box-Jenkins- ARIMA (1,1,1) log- Series 16- Lead Time > 1

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C ***********************************************************************
C
C         SERIES 16
C
C ***********************************************************************
      DO 6040 I=1,3
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=3
      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=(1.+P(1))*ALOG(X(I-1))-P(1)*ALOG(X(I-2))-P(2)*A(I-1)
      A(I)=ALOG(X(I))-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
 6000 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      SA=0.
      SAS=0.
      WRITE(6,2060)
      DO 6010 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6010 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
      K=NOB
```

Continuation Box-Jenkins- Series 16- Lead Time > 1

```
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=3,N1)
      DO 6022 I=3,N1
      F(I)=ALOG(F(I))
 6022 FI(I)=F(I)
      NN=N1+1
      DO 6020 II=NN,K
      N=II-LT+1
      DO 6021 I=N,II
      IF(I.EQ.N)GO TO 6023
      IF(I.NE.N) GO TO 6024
 6023 Z1=ALOG(X(I-1))
      A1=Z1-F(I-1)
      GO TO 6025
 6024 Z1=FI(I-1)
      A1=0.
 6025 IF((LT-2).LE.0.OR.I-2.LT.N) Z2=ALOG(X(I-2))
      IF(LT-2.GT.0.AND.I-2.GE.N) Z2=FI(I-2)
      FI(I)=(1.+P(1))*Z1-(P(1)*Z2)-(P(2)*A1)
 6021 CONTINUE
      I=II
      F(I)=FI(I)
      A(I)=ALOG(X(I))-F(I)
      IF(II.GE.N1+1) SA=A(I)+SA
      IF(II.GE.N1+1) SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      IF(I.GE.N1+1) WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
      IF(I.GE.N1+1) WRITE(8,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      WRITE(6,2060)
      J=N1+1
      SA=0.
      SAS=0.
      DO 6030 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6030 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
C         ***** FORMATS *****
 1000 FORMAT(5I5,8F8.6,F14.6)
 1010 FORMAT(8F8.0)
 1020 FORMAT(16A5)
 1030 FORMAT(5X,E14.4,2X,E14.4)
 2000 FORMAT(2X,'SERIES',I2,1X,16A5,/,2X,16A5)
```

Box_Jenkins- AR(1)- Series 18- Lead Time > 1


```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP),MEAN
C ***********************************************************************
C
C      SERIES 18
C
C ***********************************************************************
      DO 6040 I=1,14
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=2
      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      X1=ALOG(X(I-1))-MEAN
      F(I)=(P(1)*X1)+MEAN
      A(I)=ALOG(X(I))-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      XL=ALOG(X(I))
      WRITE(6,2040) I,XL,F(I),A(I),SA,SAS
 6000 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
         SA=0.
         SAS=0.
         WRITE(6,2060)
         DO 6010 I=J,K
         FL=EXP(F(I))
         AL=X(I)-FL
         SA=AL+SA
         SAS=AL**2+SAS
         WRITE(6,2040) I,X(I),FL,AL,SA,SAS
         WRITE(8,2070) I,FL,AL,X(I)
 6010    CONTINUE
         XMSE=SAS/(K-J-1)
         VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
         STD=SQRT(VAR)
         AVE=SA/(K-J)
```

Continuation Box-Jenkins Series 18- Lead Time > 1


```
              WRITE(6,2223)
              WRITE(6,2050) SA,AVE,VAR,STD,XMSE
         J=N1+1
         K=NOB
         IF(L.EQ.0) GO TO 7010
         GO TO 999
    7020 WRITE(6,2050)
         J=N1+2-L1
         N=J
         K=NOB
         SA=0.
         SAS=0.
         READ(5,1030) (F(I),A(I),I=2,N1)
         DO 6022 I=2,N1
         F(I)=ALOG(F(I))
    6022 FI(I)=F(I)
         NN=N1+1
         DO 6020 II=NN,K
         N=II-L1+1
         DO 6021 I=N,II
         IF(I.EQ.N) Z1=ALOG(X(I-1))
         IF(I.NE.N) Z1=FI(I-1)
         X1=Z1-MEAN
         FI(I)=(P(1)*X1)+MEAN
    6021 CONTINUE
         I=II
         F(I)=FI(I)
         XL=ALOG(X(I))
         A(I)=XL-F(I)
         IF(II.GE.N1+1) SA=A(I)+SA
         IF(II.GE.N1+1) SAS=A(I)**2+SAS
         IF(I.GE.N1+1) WRITE(6,2040) I,XL,F(I),A(I),SA,SAS
         IF(I.GE.N1+1) WRITE(8,2070) I,F(I),A(I),X(I)
    6020 CONTINUE
         XMSE=SAS/(K-J-1)
         VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
         STD=SQRT(VAR)
         AVE=SA/(K-J)
         WRITE(6,2223)
         WRITE(6,2050) SA,AVE,VAR,STD,XMSE
              J=N1+1
              SA=0.
              SAS=0.
              DO 6030 I=J,K
              FL=EXP(F(I))
              AL=X(I)-FL
              SA=AL+SA
              SAS=AL**2+SAS
              WRITE(6,2040) I,X(I),FL,AL,SA,SAS
              WRITE(8,2070) I,FL,AL,X(I)
    6030      CONTINUE
              XMSE=SAS/(K-J-1)
              VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
              STD=SQRT(VAR)
              AVE=SA/(K-J)
              WRITE(6,2223)
              WRITE(6,2050) SA,AVE,VAR,STD,XMSE
```

```
PROGRAM HELP        74/74    OPT=1                                FTN 4.6+460        78/04/23. 14.37.18

      C          ***** FORMATS *****
      1000 FORMAT(5I5,8F8.6,F14.6)
      1010 FORMAT(8F8.0)
      1020 FORMAT(16A5)
      1030 FORMAT(5X,E14.4,2X,E14.4)
      2000 FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
      2010 FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
          +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
          +,10X,"REGULAR AUTOREGRESSIVE PARAMETERS",F9.6,/,43X,F9.6/,44X,F9.6)
          +,/,T11,"MEAN",T44,F9.6)
      2020 FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
          +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
          + ERROR",//)
      2030 FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
          +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
          +ERROR",//)
      2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
      2050 FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
          +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
          +0X,"MEAN SQUARE ERROR ",F10.4)
      2060 FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
          +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
          + ERROR",//)
      2070 FORMAT(I5,2X,F14.4,2(2X,E14.4))
      2222 FORMAT(1H1)
      2223 FORMAT(////)
       999 STOP
           END
```

Box-Jenkins- ARIMA (1,1,0)- Series 19- Lead Time > 1

```
PROGRAM HELP        73/74    OPT=1                    FTN 4.6+460         78/04/22. 17.21.14

      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
C ***********************************************************************
C
C         SERIES 19
C
C ***********************************************************************
      DO 6040 I=1,14
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=3
      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      F(I)=((1.+P(1))*X(I-1))-(P(1)*X(I-2))
      A(I)=X(I)-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
      WRITE(8,2070) I,F(I),A(I),X(I)
 6000 CONTINUE
      XMSE=SAS/110.
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      J=N1+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=3,N1)
      DO 6022 I=3,N1
 6022 FI(I)=F(I)
      NN=N1+1
      DO 6020 II=NN,K
      N=II-LT+1
```

Continuation Box-Jenkins Series 19- Lead Time > 1

```
PROGRAM HELP        73/74    OPT=1                          FTN 4.6+460        78/04/22. 17.21.14

          DO 6021 I=N,II
          IF(I.EQ.N) Z1=X(I+1)
          IF(I.NE.N) Z1=FI(I-1)
          IF((I-2).LE.0.OR.I-2.LT.N) Z2=X(I-2)
          IF(I-2.GT.0.AND.I-2.GE.N) Z2=FI(I-2)
          FI(I)=((1.+P(1))*Z1)-(P(1)*Z2)
     6021 CONTINUE
          I=II
          F(I)=FI(I)
          A(I)=X(I)-F(I)
          IF(II.GE.N1+1) SA=A(I)+SA
          IF(II.GE.N1+1) SAS=A(I)**2+SAS
          IF(I.GE.N1+1) WRITE(6,2040) I,X(I),F(I),A(I),SA,SAS
          WRITE(6,2070) I,F(I),A(I),X(I)
          IF(I.GE.N1+1) WRITE(6,2070) I,F(I),A(I),X(I)
     6020 CONTINUE
          XMSE=SAS/(K-J)
          VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
          STD=SQRT(VAR)
          AVE=SA/(K-J)
          WRITE(6,2223)
          WRITE(6,2050) SA,AVE,VAR,STD,XMSE
C              ***** FORMATS *****
     1000 FORMAT(5I5,8F8.6,F14.6)
     1010 FORMAT(8F8.0)
     1020 FORMAT(16A5)
     1030 FORMAT(5X,E14.4,2X,F14.4)
     2000 FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
     2010 FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
         +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
         +,10X,"REGULAR AUTOREGRESSIVE PARAMETER",F8.6)
     2020 FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
         +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
         + ERROR",//)
     2030 FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
         +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
         +ERROR",//)
     2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
     2050 FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
         +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
         +0X,"MEAN SQUARE ERROR ",F10.4)
     2060 FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
         +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
         + ERROR",//)
     2070 FORMAT(I5,2X,E14.4,2(2X,E14.4))
     2222 FORMAT(1H1)
     2223 FORMAT(////)
      999 STOP
          END
```

Box-Jenkins- Multiplicative Model- (0,0,1)x(0,1,2)12 log- Series 20
Lead Time > 1

```
      PROGRAM HELP(INPUT,OUTPUT,CPUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=CP
     +UNCH)
      DIMENSION P(8),X(500),SERIES(32),A(500),F(500),FI(400)
      REAL MEAN
      READ(5,1000) NOB,N1,INDEX,NP,LT,(P(I),I=1,NP),MEAN
      READ(5,1020) (SERIES(I),I=1,32)
      READ(5,1010) (X(I),I=1,NOB)
      WRITE(6,2222)
      WRITE(6,2000) INDEX,(SERIES(I),I=1,32)
      WRITE(6,2010) NOB,N1,LT,NP,(P(I),I=1,NP)
*******************************************************************
            SERIES 20

      DO 6040 I=1,14
      A(I)=0.
 6040 F(I)=0.
      IF(LT.NE.1) GO TO 7020
      J=13

      K=N1
      L=0
 7010 IF(J.LT.75) WRITE(6,2020)
      IF(J.GE.75) WRITE(6,2030)
      IF(J.GE.75) L=1
      SA=0.
      SAS=0.
      DO 6000 I=J,K
      IF(I.EQ.13) F(I)=ALOG(X(I-12))-(P(2)*A(I-12))-(P(1)*A(I-1))
      IF(I.EQ.14) F(I)=ALOG(X(I-12))-(P(2)*A(I-12))-(P(1)*A(I-1))-(P(3)*A(I-13))
     +A(I-13))+(P(1)*P(2)*A(I-13))
      IF(I.GT.14)F(I)=ALOG(X(I-12))-(P(2)*A(I-12))-(P(3)*A(I-13))-(P(1)*
     +A(I-1))+(P(1)*P(2)*A(I-13))+(P(1)*P(2)*A(I-14))
      A(I)=ALOG(X(I))-F(I)
      SA=A(I)+SA
      SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
 6000 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      SA=0.
      SAS=0.
      WRITE(6,2060)
      DO 6010 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6010 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
```

Continuation Box-Jenkins Series 20- Lead Time > 1

```
      J=N1+1
      K=NOB
      IF(L.EQ.0) GO TO 7010
      GO TO 999
 7020 WRITE(6,2030)
      J=N1+2-LT
      N=J
      K=NOB
      SA=0.
      SAS=0.
      READ(5,1030) (F(I),A(I),I=13,N1)
      DO 6022 I=15,N1
      F(I)=ALOG(F(I))
 6022 FI(I)=F(I)
      NN=N1+1
      DO 5020 II=NN,K
      N=II-LT+1
      DO 6021 I=N,II
      IF(I.EQ.N) GO TO 6023
      IF(I.NE.N) GO TO 6024
 6023 A1=ALOG(X(I-1))-FI(I-1)
      GO TO 6025
 6024 A1=0.


 6025 IF((LT-12).LE.0.OR.I-12.LT.N) A2=A(I-12)
      IF(LT-12.GT.0.AND.I-12.GE.N) A2=0.
      IF(LT-12.LE.0.OR.I-12.LT.N) Z2=ALOG(X(I-12))
      IF(LT-12.GT.0.AND.I-12.GE.N) Z2=FI(I-12)
      IF((LT-13).LE.0.OR.I-13.LT.N) A3=A(I-13)
      IF((LT-13).GT.0.AND.I-13.GE.N)A3=0.
      IF((LT-14).LE.0.OR.I-14.LT.N) A4=A(I-14)
      IF((LT-14).GT.0.AND.I-14.GE.N)A4=0.
      FI(I)=Z2-(P(2)*A2)-(P(3)*A3)-(P(1)*A1)+(P(1)*P(2)*A3)+(P(1)*P(3)*A4)
     +4)
 6021 CONTINUE
      I=II
      F(I)=FI(I)
      A(I)=ALOG(X(I))-F(I)
      IF(II.GE.N1+1) SA=A(I)+SA
      IF(II.GE.N1+1) SAS=A(I)**2+SAS
      X1=ALOG(X(I))
      IF(I.GE.N1+1) WRITE(6,2040) I,X1,F(I),A(I),SA,SAS
      IF(I.GE.N1+1) WRITE(8,2070) I,F(I),A(I),X(I)
 6020 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
      WRITE(6,2060)
      J=N1+1
      SA=0.
      SAS=0.
      DO 6030 I=J,K
      FL=EXP(F(I))
      AL=X(I)-FL
      SA=AL+SA
      SAS=AL**2+SAS
      WRITE(6,2040) I,X(I),FL,AL,SA,SAS
      WRITE(8,2070) I,FL,AL,X(I)
 6030 CONTINUE
      XMSE=SAS/(K-J-1)
      VAR=(SAS-(SA**2/(K-J)))/(K-J-1)
      STD=SQRT(VAR)
      AVE=SA/(K-J)
      WRITE(6,2223)
      WRITE(6,2050) SA,AVE,VAR,STD,XMSE
         ***** FORMATS *****

 1000 FORMAT(5I5,8F8.6,F14.6)
 1010 FORMAT(8F8.0)
 1020 FORMAT(16A5)
 1030 FORMAT(5X,E14.4,2X,E14.4)
 2000 FORMAT(2X,"SERIES",I2,1X,16A5,/,2X,16A5)
```

Continuation Box-Jenkins Series 20- Lead Time > 1

```
2010 FORMAT(5X,"NUMBER OF OBSERVATIONS",I5,/,5X,"OBSERVATIONS USED ON M
    +ODELING ",I5,/,5X,"LEAD TIME ",I5,/,5X,"NUMBER OF PARAMETERS",I5,/
    +,10X,"REGULAR MOVING AVERAGE PARAMETER",F8.6,/,10X,"SEASONAL MOVING AVERAGE
    +G AVERAGE PARAMETERS",F8.6,/,44X,F8.6)
2020 FORMAT(//,30X,"**** INITIALIZATION PHASE ****",//,2X,"PERIOD",5X,"OBSERVAT
    +OBSERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM.ERROR",5X,"SUM OF SQ. ERROR"
    +ERROR",//)
2030 FORMAT(//,30X,"**** FORECASTING PHASE ****",//,2X,"PERIOD",5X,"OBSE
    +ERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ. ERROR
    +ERROR",//)
2040 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
2050 FORMAT(10X,"SUM OF ERROR ",F12.4,/,10X,"AVERAGE ERROR ",F10.4,/,10
    +X,"VARIANCE OF ERROR ",F10.4,/,10X,"STANDARD DEVIATION ",F10.4,/,1
    +0X,"MEAN SQUARE ERROR ",F10.4)
2060 FORMAT(1H1,30X,"**** ANTILOGS OF ABOVE ****",//,2X,"PERIOD",5X,"OB
    +SERVATION",5X,"FORECAST",5X,"ERROR",5X,"CUM. ERROR",5X,"SUM OF SQ.
    + ERROR",//)
2070 FORMAT(I3,2X,E14.4,2(2X,E14.4))
2222 FORMAT(1H1)
2223 FORMAT(////)
 999 STOP
     END
```

APPENDIX C

COMBINATIONS OF FORECASTS

Instructions for the use of COMBI

PURPOSE: To obtain a composite forecast using the method developed on Chapter 3.

INPUT PREPARATION:
Refer to listing, program is self-explanatory

FLOW CHART:

Input forecasts and forecast errors for
each forecasting method
Input parameters for combination formulas

↓

1 → Input how many forecasting methods will be combined

↓

Compute statistics for each method in the combinations
(Mean error, variance of error, mean square error)

↓

Compute correlation coefficient

↓

Compute covariance and inverse of covariance matrix

↓

Compute weights

↓

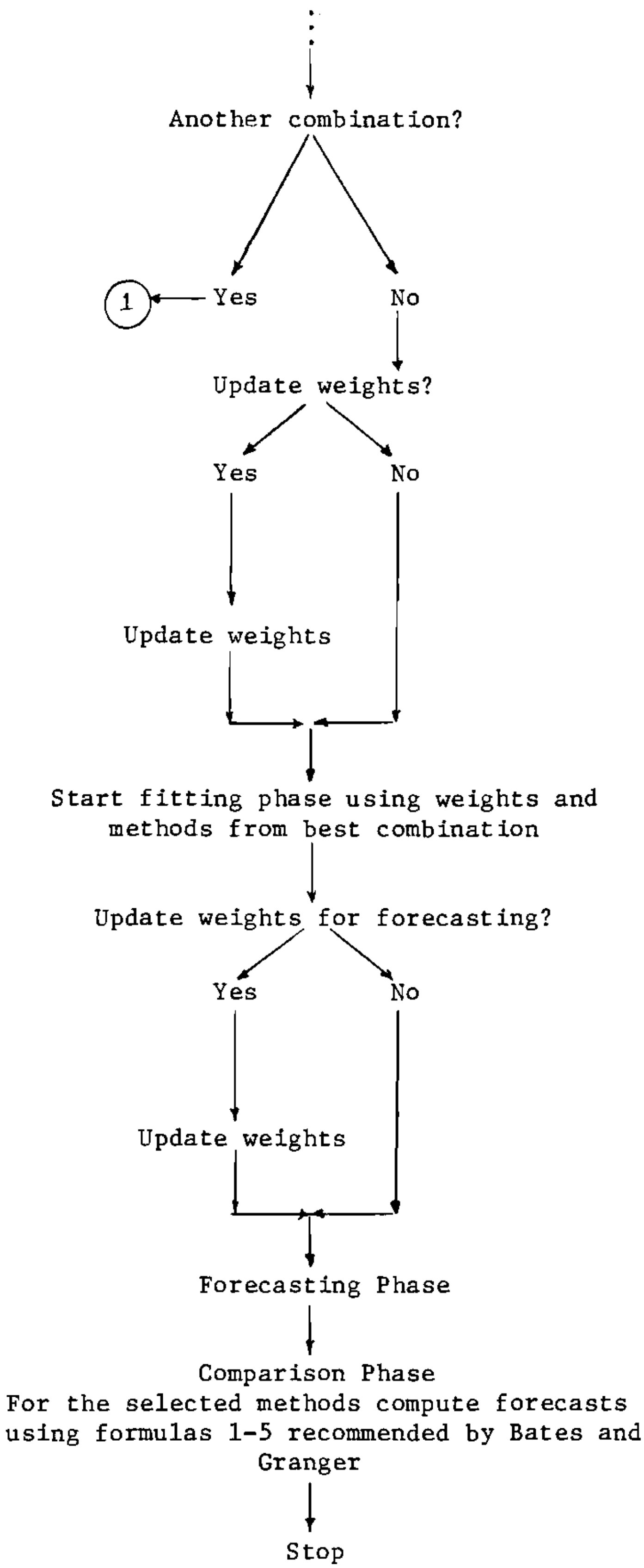
Compute variance for combination

↓

Save best combination so far

↓

⋮


Another combination?
1
Yes
No
Update weights?
Yes
No
Update weights
Start fitting phase using weights and methods from best combination
Update weights for forecasting?
Yes
No
Update weights
Forecasting Phase
Comparison Phase
For the selected methods compute forecasts using formulas 1-5 recommended by Bates and Granger
Stop

Listing for Computer Program COMBI

```
      PROGRAM COMBI(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,TAPE8)
      DIMENSION X(400,5),SERIES(16),METHOD(5),XVAR(5),STD(5),R(5,5),B(5)
     +,RX(5,5),D(5),TI(5),Y(400,5)
      DIMENSION DATA(5),RINV(5,5),DEN(5),F(500),DV(500),E(500)
      DIMENSION W(5),WKAREA(40),SW(5),INP(5)
      DIMENSION E1(5),FF(5),SFF(5),SSFF(5)
      DIMENSION C(5,5),CINV(5,5),SC(5),SWC(5),WCINV(5,5)
      DIMENSION WE(5,5),E2(5),WC(5,5)
      DIMENSION CVARE(5),CSVARE(5),CXMSE(5),SR(5,5)
      INTEGER T,TT,FM,TIME,TM1
      REAL KW(5,5),MSE(5)
      INTEGER OPTION(12),BLANK(5),TITLE(5),COMB(5),SN,SM(5)
      DATA METHOD/8HMETHOD 1,8HMETHOD 2,8HMETHOD 3,8HMETHOD 4,8HMETHOD 5/
     +/
      DATA BLANK/10H          ,10H          ,10H          ,10H          ,
     +,10H          /
      DATA ERRORS/6HERRORS/
C    ****************************************************************************
C     THESIS  A STATISTICAL APPROACH TO THE COMBINATION OF FORECASTS
C             -----------------------------------------------------
C
C             SIMULATION STUDY
C              'OUR INTEREST IS IN CASES IN WHICH TWO (OR MORE) FORECASTS
C               HAVE BEEN MADE OF THE SAME EVENT. TYPICALLY, THE REACTION
C               OF MOST STATISTICIANS AND BUSINESSMEN WHEN THIS OCCURS IS
C               TO ATTEMPT TO DISCOVER WHICH IS THE BETTER (OR BEST)
C               FORECAST  THE BETTER FORECAST IS THEN ACCEPTED AND USED,
C               THE OTHER(S) BEING DISCARDED.'  J.M. BATES AND C.W.J.
C               GRANGER- 'THE COMBINATION OF FORECASTS'(1969). OPER.RES.Q.
C               20 451-468.
C
C              'BATES AND GRANGER HAVE SUGGESTED THAT THE DISCARDED
C               FORECAST(S) WILL IN ALMOST ALL CASES CONTAIN ADDITIONAL
C               INFORMATION NOT PRESENT IN THE ADOPTED FORECAST.
C               CONSEQUENTLY THEY PROPOSED THAT A LINEAR COMBINATION
C               OF FORECASTS, USING A MINIMUM VARIANCE CRITERION TO
C               DETERMINE THE WEIGHTS BE USED AS A COMPOSITE FORECAST'.
C               J.P. DICKINSON- (1973)-'SOME STATISTICAL RESULTS IN THE
C               COMBINATION OF FORECASTS'-OPER.RES. Q.24 253-260.
C
C
C             GENOVEVA CRUZ                     SEPT. 1977
C
C                CARD INPUT FORMAT
C                -----------------
C                HEADER CARD 1
C                -------------
C                  COL        DESCRIPTION
C                   1- 5      N          = NO. OF METHODS TO BE COMBINED N.LT.5
C                   6-10      N1         = NO. OF DATA POINTS IN ORIGINAL SERIES
C                                            N1.LT.500
C                  11-15      NF         = NO. OF FORECASTS
C                  21-25      OPTION(1) = 0 IF DATA LISTING IS DESIRED AS PART
C                                            OF OUTPUT
C                  26-30      OPTION(2) = 0 IMPLIES SUMMARY FOR EACH STAGE
C                  31-35      OPTION(3) = 0 IMPLIES COVARIANCE MATRIX FOR EACH
```

```
C                                   STAGE IS DESIRED AS PART OF OUTPUT
C              36-40     OPTION(4) = 0 IMPLIES INV. OF COV. MATRIX ON OUTPUT
C              41-45     OPTION(5) = 0 IMPLIES WEIGHTS AT EACH STAGE ARE DES
C              46-50     OPTION(6) = 0 IMPLIES ALPHA IS NOT USED TO UPDATE
C                                   WEIGHTS
C              51-55     OPTION(7) = 0 IMPLIES DIFFERENT METHODS OF
C                                   COMBINATIONS USED FOR COMPARISON
C                                   PURPOSES
C              56-60     OPTION(8) = 0 IMPLIES COMBINED FORECASTS FOR JUST
C                                   FORMULA 6 ARE DESIRED
C                                   USE 1 IF OPTION(7)=0 BECAUSE
C                                   SOME CALCULATIONS WILL BE
C                                   DUPLICATED
C              61-70     OPTION(9) = 0 PLOT OF FORECASTS (FOR FORMULA (6))
C                                     DESIRED AS PART OF OUTPUT ON
C                                     FORECASTING PHASE
C
C            HEADER CARD 2
C            -------------
C              COL       DESCRIPTION
C               1-10     DATA(I)    = ALPHANUMERIC  TITLE FOR METHOD I
C                                     ( EX. REGRESSION, ADAPTIVE, SMOOTHING)
C
C            HEADER CARD 3 ( DATA CARDS FOR METHOD I)
C            -------------
C              COL       DESCRIPTION
C               1- 3     K          = INDEX FOR DATA VALUE (1,...,N1)
C               6-20     Y(K,I)     = FORECAST FROM METHOD I, PERIOD K
C              21-34     X(K,I)     = FORECAST ERROR
C                        FOR EACH METHOD INCLUDE CARD 2 AND 3 IN SAME ORDER,
C                        AS REQUIRED
C                        WHEN ALL THE DATA CARDS ARE FINISHED, INCLUDE NEXT
C                        HEADING CARD.
C
C            HEADING CARD
C            ------------
C              COL       DESCRIPTION
C               1- 6     SERIES(K) = ALPHANUMERIC TITLE FOR THE
C                                     SERIES UNDER STUDY. TO BE INCLUDED
C                                     AS PART OF OUTPUT. THE TITLE CANNOT
C                                     EXCEED 80 COLUMNS.
C
C            HEADING CARD
C            ------------
C              COL       DESCRIPTION(F5.0)
C               1- 5     NU         = CONTROL ON SUMMATION FOR METHOD 1
C                                     NU > 1
C               6-10     BETA       = SMOOTHING CONSTANT FOR COMB. METHOD 2
C                                     BETA >= 1
C              11-15     GAMMA      = WEIGHT TO BE USED ON COMB. METHOD 3
C                                     0 < GAMMA < 1
C              16-20     ZETA       = WEIGHT TO BE USED ON COMB. METHOD 4
C                                     ZETA >= 1
C              21-25     XI         = WEIGHT TO BE USED IN FORMULA 5
C                                     XI>= 1
C              26-30     ALPHA      = SMOOTHING CONSTANT FOR UPDATING WEIGHTS
C                                     0< ALPHA <1
C
C            HEADING CARD (FOR EACH ITERATION)
C            ------------
C              COL       DESCRIPTION
C               1- 5     N          = NO. OF METHODS TO BE COMBINED IN THIS
C                                     STEP N.LT.5
C               6-10     COMB(1)    = INDEX FOR FIRST METHOD TO BE COMBINED
C              11-15     COMB(2)    = INDEX FOR SECOND
C              .         .            .
C              .         .            .
C  **************************************************************************
```

```
      READ(5,1000) N,N1,NF,(OPTION(I),I=1,11)
      N2=N1+NF
      DO 10 I=1,N
      READ(5,1030) DATA(I)
      READ(5,1010)(K,Y(K,I),X(K,I),DV(K))
      INP(I)=K
   12 READ(5,1010)(K,Y(K,I),X(K,I),DV(K))
      IF(K.EQ.N2) GO TO 10
      GO TO 12
   10 CONTINUE
C     READ(5,1012) (DV(K),K=1,N2)
      READ(5,1020)(SERIES(K),K=1,16)
      READ(5,1021) ALPHA,BETA,GAMMA,ZETA,NU,XI
      WRITE(6,2000)
      WRITE(6,2001)
      WRITE(6,2010)(SERIES(I),I=1,16)
      IF(OPTION(1).EQ.0)WRITE(6,2020) ((METHOD(I),ERRORS),I=1,N)
      WRITE(6,2021)(DATA(I),I=1,N)
      IF(OPTION(1).NE.0) GO TO 31
      DO 11 K=1,N2
   11 WRITE(6,2030) K,DV(K),(Y(K,I),X(K,I),I=1,N)
      SVAR=99999.
C
C           TITLES FOR SUMMARY OF STATISTICAL VALUES
C
C   ****************************************************************************
   31 READ(5,1000) N,(COMB(I),I=1,N)
      IF(EOF(5)) 289,32
   32 IF(OPTION(2).EQ.0) WRITE(6,2031) N1,(DATA(COMB(I)),I=1,N)
C           N HERE IS THE NUMBER OF METHODS TO BE COMBINED
C             ON THIS STEP
C            INITIALIZATION
      M=N
      SMINV=9999999.
      I1=1
      DO 100 J=1,M
      II=COMB(J)
      IF(I1.LT.INP(II)) I1=INP(II)
      JJ=J+1
      IF(JJ.LE.M) GO TO 101
      R(J,J)=0.0
  102 B(J)=0.
      TI(J)=0.
      GO TO 100
  101 R(J,JJ)=0.0
      R(JJ,J)=0.0
      GO TO 102
  100 CONTINUE
      L=0
C        DATA ARE ALREADY IN CORE
      DO 108 J=1,M
      JJ=COMB(J)
      DO 107 I=I1,N1
      L=L+1
  107 TI(J)=TI(J)+X(I,JJ)
  108 CONTINUE
      IF (OPTION(2).EQ.0) WRITE(6,2044)(TI(J),J=1,M)
      DO 109 J=1,M
      FN=N1-INP(J)
      E1(J)=0.
  109 TI(J)=TI(J)/FN
      IF (OPTION(2).EQ.0) WRITE(6,2043) (TI(J),J=1,M)
      FN=N1-I1
      DO 115 I=I1,N1
      DO 110 J=1,M
      JJ=COMB(J)
```

```
      D(J)=X(I,JJ)-TI(J)
      B(J)=B(J)+D(J)
      E1(J)=X(I,JJ)**2+E1(J)
  110 CONTINUE
      DO 115 J=1,M
      DO 115 K=1,J
      R(J,K)=R(J,K)+D(J)*D(K)
  115 CONTINUE
      IF(OPTION(2).EQ.0) WRITE(6,2045) (B(J),J=1,M)
C          ADJUST SUMS OF CROSS-PRODUCTS OF DEVIATIONS
C          FROM TEMPORARY MEANS
      DO 210 J=1,M
      DO 210 K=1,M
      R(J,K)=R(J,K)-B(J)*B(K)/FN
  210 CONTINUE
C          CALCULATE CORRELATION COEFFICIENTS
      DO 220 J=1,M
  220 STD(J)=SQRT(ABS(R(J,J)))
      DO 230 J=1,M
      DO 230 K=1,J
      IF(STD(J)*STD(K)) 225,222,225
  222 R(J,K)=0.
      GO TO 230
  225 RX(J,K)=R(J,K)/(STD(J)*STD(K))
      R(J,K)=R(J,K)/(FN-1)
      R(K,J)=R(J,K)
  230 CONTINUE
      FM=FN
C            CALCULATE STANDARD DEVIATION
      FN=SQRT(FN-1.0)
      DO 240 J=1,M
      STD(J)=STD(J)/FN
      MSE(J)=E1(J)/(FM-1)
      XVAR(J)=STD(J)**2
 240  IF(XVAR(J).LT.SMINV) SMINV=XVAR(J)
      IF(OPTION(2).EQ.0) WRITE(6,2047) (XVAR(J),J=1,M)
      IF (OPTION(2).EQ.0) WRITE(6,2046) (STD(J),J=1,M)
      IF(OPTION(2).EQ.0) WRITE(6,2042)(MSE(J),J=1,M)
      IF(OPTION(2).EQ.0) WRITE(6,2041) FM
      IF(OPTION(2).EQ.0)WRITE(6,2048)(DATA(COMB(J)),J=1,N)
      DO 242 J=1,N
      IF(OPTION(2).EQ.0) WRITE(6,2049)DATA(COMB(J)),(RX(J,K),K=1,J)
  242 CONTINUE
      IF(OPTION(3).EQ.0) WRITE(6,2050)(DATA(COMB(J)),J=1,N)
      DO 243 J=1,N
      R(J,J)=STD(J)**2
      IF (OPTION(3).EQ.0) WRITE(6,2049)DATA(COMB(J)),(R(J,K),K=1,N)
  243 CONTINUE
      DO 260 I=1,N
      DO 260 J=I,N
      R(I,J)=R(J,I)
  260 CONTINUE
C
C             CALCULATE INVERSE OF MATRIX USING SUBROUTINE FROM
C             LIBRARY IMSL-  INTERNATIONAL MATH SCIENCE LIBRARY
C
      CALL LINV2F(R,N,5,RINV,10,WKAREA,IER)
      IF(OPTION(4).EQ.0) WRITE(6,2060) (DATA(COMB(J)),J=1,M)
      DO 261 I=1,N
      IF(OPTION(4).EQ.0) WRITE(6,2049)DATA(COMB(I)),(RINV(I,J),J=1,N)
 261  CONTINUE
C
C            COMPUTE VECTOR OF WEIGHTS
C
      DENOM=0.
      DO 270 I=1,N
```

```
      DEN(I)=0.
      J=0
 271  J=J+1
      IF(J.GT.N) GO TO 272
      DEN(I)=RINV(I,J)+DEN(I)
      GO TO 271
 272  DENOM=DEN(I)+DENOM
  270 CONTINUE
C
C           COMPUTE WEIGHTS
C
      SUMW=0.
      DO 280 I=1,N
      W(I)=DEN(I)/DENOM
  280 SUMW=W(I)+SUMW
      IF(OPTION(4).EQ.0) WRITE(6,2069)
      IF(OPTION(4).EQ.0)WRITE(6,2070)(W(I),I=1,N)
C
C           COMPUTE VARIANCE OF THE COMBINED FORECAST ERROR
C
      VAR=1./DENOM
      NP1=N+1
      PERC=((SMINV-VAR)/SMINV)*100.
      IF(NP1.EQ.6) WRITE(6,2071) N,(DATA(COMB(I)),I=1,N),VAR,PERC
      IF(NP1.LT.6) WRITE(6,2071) N,(DATA(COMB(I)),I=1,N),(BLANK(J),J=NP1,5),VAR
     +,5),VAR,PERC
      IF(VAR.LT.SVAR) GO TO 287
      GO TO 31
  287 SVAR=VAR
      SN=N
      DO 288 I=1,N
      SW(I)=W(I)
      SM(I)=COMB(I)
      DO 288 II=1,I
      SR(I,II)=R(I,II)
  288 CONTINUE
      GO TO 31
 289  IF(SN.LT.5)I2=SN+1
      IF(SN.EQ.5)I2=5
      IF(SN.LT.5) WRITE(6,2074) SN,(DATA(SM(I)),I=1,SN),(BLANK(J),J=I2,5),SVAR
     +),SVAR
      IF(SN.EQ.5) WRITE(6,2074) SN,(DATA(SM(I)),I=1,5),SVAR
C                    START FORECASTING PHASE
C                RELOCATE VARIABLES ON SAVE LOCATIONS
      IF(OPTION(8).EQ.0) WRITE(6,2072)
      N=SN
      IH=1
      CE=0
      SES=0.
      DO 293 I=1,N
      IF(INP(SM(I)).GT.IH) IH=INP(SM(I))
      W(I)=SW(I)
      DO 293 L=I,N
      R(I,L)=SR(I,L)
      R(L,I)=R(I,L)
  293 CONTINUE
      DO 4021 T=IH,N1
      F(T)=0.
      DO 4020 I=1,N
      J=SM(I)
4020  F(T)=W(I)*Y(T,J)+F(T)
      E(T)=DV(T)-F(T)
      CE=E(T)+CE
      SES=E(T)*E(T)+SES
4021  IF(OPTION(8).EQ.0) WRITE(6,2073) T,DV(T),F(T),E(T),CE,SES
      AE=CE/NF
```

```
      FN=FM+1
      VARE=(SES-(CE**2)/FN)/(FN-1)
      SVARE=SQRT(VARE)
      XMSE=SES/(FN-1)
      IF(OPTION(8).EQ.0) WRITE(6,2080) CE,AE,VARE,SVARE,XMSE
      IF(OPTION(6).EQ.0.AND.OPTION(8).EQ.0) WRITE(6,2075) ALPHA
      IF(OPTION(8).EQ.0) WRITE(6,2077)
      IF(OPTION(6).EQ.1.AND.OPTION(8).EQ.0) WRITE(6,2076)
      CE=0.
      SES=0.
      N3=N1+1
      N2=N1+NF
      DO 4000 T=N3,N2
      F(T)=0.
      IF(OPTION(6).EQ.1) GO TO 9998
      DO 4001 I=1,N
      DO 4001 II=I,N
      R(I,II)=ALPHA*X(T,SM(I))*X(T,SM(II))+(1.-ALPHA)*R(I,II)
      R(II,I)=R(I,II)
4001  CONTINUE
C               CALCULATE INVERSE OF UPDATED COVARIANCE MATRIX
      CALL LINV2F(R,N,5,RINV,10,WKAREA,IER)
C               COMPUTE UPDATED VECTOR OF WEIGHTS
      DENOM=0.
      DO 283 II=1,N
      DEN(II)=0.
      J=0
 284  J=J+1
      IF(J.GT.N) GO TO 285
      DEN(II)=RINV(II,J)+DEN(II)
      GO TO 284
 285  DENOM=DEN(II)+DENOM
 283  CONTINUE
      DO 286 II=1,N
      W(II)=DEN(II)/DENOM
 286  CONTINUE
 281  DO 291 J=1,N
      JJ=SM(J)
      F(T)=W(J)*Y(T,JJ)+F(T)
 291  CONTINUE
      E(T)=DV(T)-F(T)
      CE=E(T)+CE
      SES=E(T)*E(T)+SES
      IF(OPTION(8).EQ.0) WRITE(6,2073) T,DV(T),F(T),E(T),CE,SES
4000  CONTINUE
      AE=CE/NF
      VARE=(SES-(CE**2)/NF)/(NF-1)
      SVARE=SQRT(VARE)
      XMSE=SES/(NF-1)
      IF(OPTION(8).EQ.0) WRITE(6,2080) CE,AE,VARE,SVARE,XMSE
      IF(OPTION(7).EQ.1) GO TO 9999
 9998 IF(OPTION(6).EQ.0) WRITE(6,2082) ALPHA,NU,BETA,GAMMA,ZETA,XI
C  ****************************************************************************
C
C          START FORECASTING PHASE
C
C              COMPARISON PHASE
C
C  ****************************************************************************
C
C              WRITE TITLES
      WRITE(6,2083)
      WT=0.
      CEAVE=0.
      SEAVES=0.
      SRE2=0.
```

```
      DO 292 L=1,5
      E1(L)=0.
      E2(L)=0.
      FF(L)=0.
      SFF(L)=0.
      SSFF(L)=0.
      KW(3,L)=1./N
      DO 292 LL=L,5
      WE(L,LL)=0.
      C(L,LL)=0.
 292  CONTINUE
      DO 4002 T=IH,N1
      WT=ZETA**T+WT
      DO 4002 I=1,N
      J=SM(I)
      E2(I)=(XI**T)*(X(T,J)**2)+E2(I)
      WRITE(8,5003) E2(I)
 5003 FORMAT(2X,'E2(I)=',F12.6)
      IF(E2(I).NE.0)SRE2=(1./E2(I))+SRE2
      WRITE(8,5004) SRE2
 5004 FORMAT(2X,'SRE2=',F12.6)
      DO 4002 II=I,N
      JJ=SM(II)
      WE(I,II)=(ZETA**T)*(X(T,J)*X(T,JJ))+WE(I,II)
      WRITE(8,5005) WE(I,II)
 5005 FORMAT(2X,'WE(I,II)',F12.6)
4002  CONTINUE
C                T IS NOW CONTROL FOR TIME FROM ORIGIN
      DO 4003 T=N3,N2
      FAVE=0.
      II=T-NU
      IT=T-1
      SRE1=0.
C                TT IS CONTROL FOR TIME ON COMBINATION METHOD 1
      DO 4004 TT=II,IT
      DO 4004 I=1,N
      J=SM(I)
      E1(I)=X(TT,J)**2+E1(I)
      DO 4004 III=I,N
      JJ=SM(III)
      C(I,III)=X(TT,J)*X(TT,JJ)+C(I,III)
      WRITE(8,5006) C(I,III),E1(I)
 5006 FORMAT(2X,'C(I,III)=',F12.6,2X,'E1(I)=',F12.6)
4004  CONTINUE
      DO 4005 I=1,N
      IF(E1(I).NE.0) SRE1=(1./E1(I))+SRE1
      WRITE(7,5007) SRE1
 5007 FORMAT(2X,'SRE1=',F12.6)
      DO 4005 III=I,N
      IF(BETA.NE.0.) C(I,III)=(1./BETA)*C(I,III)
      C(III,I)=C(I,III)
      IF(WT.NE.0) WC(I,III)=WE(I,III)/WT
      WRITE(7,5008) C(I,III),WC(I,III)
 5008 FORMAT(2X,'C(I,III)=',F12.6,2X,'WC(I,III)=',F12.6)
      WC(III,I)=WC(I,III)
4005  CONTINUE
C           COMPUTE INVERSE OF MATRIX C (FOR FORMULA 1)
      CALL LINV2F(C,N,5,CINV,5,WKAREA,IER)
C           COMPUTE INVERSE OF MATRIX WC (FOR FORMULA 4)
      CALL LINV2F(WC,N,5,WCINV,5,WKAREA,IER)
      SCT=0.
      SWCT=0.
      DO 4006 I=1,N
      SC(I)=0.
      SWC(I)=0.
      DO 4006 L=1,N
```

```
C          ADD ROWS ON INVERSE MATRIX
      SC(I)=CINV(I,L)+SC(I)
      SWC(I)=WCINV(I,L)+SWC(I)
C         ADD ELEMENTS ON INVERSE MATRIX
      SCT=CINV(I,L)+SCT
      SWCT=WCINV(I,L)+SWCT
      WRITE(7,5009) SC(I),SWC(I),SCT,SWCT
4006  CONTINUE
 5009 FORMAT('SC(I)=',F12.6,'SWC(I)=',F12.6,'SCT=',F12.6,'SWCT=',F12.6)
C          ALL COMPUTATIONS READY, COMPUTE WEIGHTS
      DO 4007 I=1,N
      IF(SCT.NE.0) KW(1,I)=SC(I)/SCT
      IF(E1(I).NE.0.AND.SRE1.NE.0.) KW(2,I)=(1./E1(I))/SRE1
      KW(3,I)=GAMMA*KW(3,I)+(1.-GAMMA)*KW(2,I)
      IF(KW(3,I).LT.0.00001) KW(3,I)=1./N
 5002 FORMAT(5F10.6)
      IF(SWCT.NE.0.) KW(4,I)=SWC(I)/SWCT
      IF(SWCT.EQ.0.OR.SWC(I).EQ.0) KW(4,I)=1.0
      IF(E2(I).NE.0.AND.SRE2.NE.0) KW(5,I)=(1./E2(I))/SRE2
      IF(KW(5,I).LT.0.0001) KW(5,I)=1./N
C          INITIATE CONDITIONS FOR NEXT ITERATION ON T
4007  CONTINUE
      WRITE(7,5002) ((KW(IA,IB),IA=1,5),IB=1,N)
      SRE2=0.
      DO 4008 I=1,N
      J=SM(I)
      E1(I)=0.
      E2(I)=E2(I)+X(T,J)**2
      IF(E2(I).NE.0) SRE2=(1./E2(I))+SRE2
      FAVE=Y(T,J)+FAVE
      DO 4008 II=I,N
      JJ=SM(II)
      WE(I,II)=(XI**T)*(X(T,J)*X(T,JJ))+WE(I,II)
      C(I,II)=0.
      C(II,I)=0.
4008  CONTINUE
      DO 4009 I=1,N
      J=SM(I)
C            L IS THE CONTROL FOR EACH COMBINATION METHOD
      DO 4009 L=1,5
      TI(L)=0.
      FF(L)=KW(L,I)*Y(T,J)+FF(L)
 4009 CONTINUE
      FAVE=FAVE/N
      WRITE(6,2084) T,DV(T),FAVE,(FF(L),L=1,5),F(T)
      DO 2092 L=1,5
      EE=DV(T)-FF(L)
      SFF(L)=EE+SFF(L)
      SSFF(L)=EE**2+SSFF(L)
      FF(L)=0.
 2092 CONTINUE
      EAVE=DV(T)-FAVE
      CEAVE=EAVE+CEAVE
      SEAVES=EAVE*EAVE+SEAVES
      WT=ZETA**T+WT
      WRITE(7,5010) WT
 5010 FORMAT(2X,'WT=',F12.6)
4003  CONTINUE
C   ****************************************************************************
C               COMPUTE MEAN,VARIANCE, STD. DEV., MEAN SQUARE ERROR.
C                    FOR METHOD OF AVERAGES
      EVARE=(SEAVES-(CEAVE**2)/NF)/(NF-1)
      EAVE=CEAVE/NF
      ESVARE=SQRT(EVARE)
      EXMSE=SEAVES/(NF-1)
```

```
C                   FOR BATES AND GRANGER'S METHODS- FORMULA 1-5
      DO 3004 L=1,5
      TI(L)=SFF(L)/NF
      CVARE(L)=(SSFF(L)-(SFF(L)**2)/NF)/(NF-1)
      CSVARE(L)=SQRT(CVARE(L))
      CXMSE(L)=SSFF(L)/(NF-1)
 3004 CONTINUE
      WRITE(6,2085) EAVE,(TI(L),L=1,5),AE
      WRITE(6,2086) EVARE,(CVARE(L),L=1,5),VARE
      WRITE(6,2087) ESVARE,(CSVARE(L),L=1,5),SVARE
      WRITE(6,2088) EXMSE,(CXMSE(L),L=1,5),XMSE
9999  IF(OPTION(9).EQ.1) GO TO 99990
      CALL PLOT(DV,F,N2,N2,100,1,1,1)
C  ***********************************************************************
C                   FORMATS
 1000 FORMAT(16I5)
 1010 FORMAT(I3,2X,E14.4,2(2X,E14.4))
 1020 FORMAT(16A5)
 1021 FORMAT(4F5.0,I5,F5.0)
 1030 FORMAT(8A10)
 2000 FORMAT(1H1,2X,'THESIS: A STATISTICAL APPROACH TO THE COMBINATION O
     +F FORECASTS',/,10X,55('.'),/,10X,'GENOVEVA CRUZ',33X,'FALL 1977')
 2001 FORMAT(/,10X,55('.'),/,10X,'A LINEAR COMBINATION OF FORECASTS CAN
     +GIVE A SMALLER',/,10X,'ERROR VARIANCE THAN ANY OF THE INDIVIDUAL F
     +ORECASTING',/,10X,'METHODS...')
 2010 FORMAT(/,10X,16A5)
 2020 FORMAT(//,1X,'DATA LISTING:',//,17X,5(A10,2X,A6,2X))
 2021 FORMAT(1X,'PERIOD',3X,'DATA',2X,5(A10,10X),/1X,115('.'),/)
 2030 FORMAT(I5,11F10.4)
 2031 FORMAT(1H1,//,2X,'SUMMARY OF STATISTICAL VALUES:(FIRST',I5,' POINTS
     +S OF DATA CONSIDERED)',/,2X,'METHOD:',22X,5(A10,5X))
 2040 FORMAT(1H1)
 2041 FORMAT(//,2X,'NUMBER OF ERRORS USED IN COMBINATION:', I5)
 2042 FORMAT(/,2X,'(6) MEAN SQUARE ERROR:',2X,5(F12.4,3X))
 2043 FORMAT(/,2X,'(2) MEANS:',14X,5(F12.4,3X))
 2044 FORMAT(/,2X,'(1) SUM OF ERRORS:',6X,5(F12.4,3X))
 2045 FORMAT(/,2X,'(3) SUM(ERROR-MEAN):',4X,5(F12.4,3X))
 2046 FORMAT(/,2X,'(5) STANDARD DEVIATION:',1X,5(F12.4,3X))
 2047 FORMAT(/,2X,'(4) VARIANCE:',11X,5(F12.4,3X))
 2048 FORMAT(//,2X,'CORRELATION COEFFICIENTS:',/,20X,5(A8,8X))
 2049 FORMAT(2X,A8,6X,5(F12.4,4X))
 2050 FORMAT(//,2X,'COVARIANCE MATRIX:',/,20X,5(A8,8X))
 2060 FORMAT(//,2X,'INVERSE OF COVARIANCE MATRIX:',/,20X,5(A8,8X))
 2069 FORMAT(//,2X,'WEIGHT VECTOR FOR COMBINED FORECASTS')
 2070 FORMAT(5(2X,E14.4,/))
 2071 FORMAT(//,2X,'COMBINATION OF ',I4,' METHODS ',4(A10,'  '),A10,' ERROR
     +ROR VARIANCE= ',F10.4,/,88X,' % IMPROVEMENT OVER MIN. VARIANCE:',F6.2)
     +6.2)
 2072 FORMAT(1H1,30X,'**** INITIALIZATION PHASE ****',//,2X,'PERIOD',5X,
     +'OBSERVATION',5X,'FORECAST',5X,'ERROR',5X,'CUM. ERROR',5X,'SUM OF
     +SQ. ERROR',//)
 2073 FORMAT(2X,I6,5X,F10.4,3X,F10.4,2X,F10.4,4X,F10.4,5X,F14.4)
 2074 FORMAT(/,2X,'BEST COMBINATION OCCURS FOR',I4,' METHODS ',5(A10,2X)
     +,/,2X,'FOR WHICH VARIANCE OF THE COMBINED FORECAST ERROR IS ',F14.
     +4)
 2075 FORMAT(/,2X,'WEIGHTS UPDATED USING ALPHA=', F6.2)
 2076 FORMAT(/,2X,'NO UPDATE OF WEIGHTS USED')
 2077 FORMAT(1H1,30X,'**** FORECASTING PHASE ****',//,2X,'PERIOD',5X,'OB
     +SERVATION',5X,'FORECAST',5X,'ERROR',5X,'CUM. ERROR',5X,'SUM OF SQ.
     + ERROR',//)
 2080 FORMAT(1H0,2X,'SUM OF FORECAST ERRORS=',F10.4,5X,'AVERAGE FORECAST
     + ERROR=',F10.4,5X,'VARIANCE=',F10.4,5X,'STANDARD DEVIATION=',F10.4,
     +,/,2X,'MEAN SQUARE ERROR=',E14.6)
 2082 FORMAT(/,2X,'WEIGHTS UPDATED USING ALPHA=',F6.2,/,2X,'FORMULA 1 US
     +ES NU=  ',I8,/,2X,'FORMULA 2 USES BETA=',F8.4,/,2X,'FORMULA 3 USE
     +GAMMA=',F8.4,/,2X,'FORMULA 4 USES ZETA=',F8.4,/,2X,'FORMULA 5 USES
```

```
     + XI=',F8.4)
 2083 FORMAT(1H1,//,40X,'****  COMBINED FORECASTS  ****',//,2X,'PERIOD',
     +5X,'DATA',7X,'AVERAGE',5X,'FORMULA 1',3X,'FORMULA 2',3X,'FORMULA 3
     +',3X,'FORMULA 4',3X,'FORMULA 5',3X,'FORMULA 6(*)',/,2X,106('.'))
 2084 FORMAT(2X,I5,1X,8(2X,F10.4))
 2085 FORMAT(/,2X,'MEAN ERROR=',7X,8(2X,F10.4))
 2086 FORMAT(/,2X,'ERROR VARIANCE=',6X,F9.4,7(2X,F10.4))
 2087 FORMAT(/,2X,'STD.DEV.=',12X,F9.4,7(2X,F10.4))
 2088 FORMAT(/,2X,'MSE ERROR=',8X,8(2X,F10.4))
C  ***************************************************************************
99990 STOP
      END
C
      SUBROUTINE LINV2F (A,N,IA,AINV,IDGT,WKAREA,IER)                   LI2F0010
C-LINV2F---------S--------LIBRARY 3--------------------------------------LI2F0030
C   FUNCTION            - INVERSION OF A MATRIX - FULL STORAGE MODE -   LI2F0050
C                           HIGH ACCURACY SOLUTION                      LI2F0060
C   USAGE               - CALL LINV2F (A,N,IA,AINV,IDGT,WKAREA,IER)     LI2F0070
C   PARAMETERS   A      - INPUT MATRIX OF DIMENSION N BY N CONTAINING   LI2F0080
C                           THE MATRIX TO BE INVERTED.                  LI2F0090
C                N      - ORDER OF A. (INPUT)                           LI2F0100
C                IA     - NUMBER OF ROWS IN THE DIMENSION STATEMENT     LI2F0110
C                           FOR A AND AINV IN THE CALLING PROGRAM.      LI2F0120
C                           (INPUT)                                     LI2F0130
C                AINV   - OUTPUT MATRIX OF DIMENSION N BY N CONTAINING  LI2F0140
C                           THE INVERSE OF A. A AND AINV MUST OCCUPY    LI2F0150
C                           SEPARATE CORE LOCATIONS.                    LI2F0160
C                IDGT   - INPUT OPTION.                                 LI2F0170
C                         IF IDGT IS GREATER THAN 0, THE ELEMENTS OF A  LI2F0180
C                           ARE ASSUMED TO BE CORRECT TO IDGT DECIMAL   LI2F0190
C                           DIGITS AND THE ROUTINE PERFORMS AN ACCURACY LI2F0200
C                           TEST.                                       LI2F0210
C                         IF IDGT EQUALS 0, THE ACCURACY TEST IS        LI2F0220
C                           BYPASSED.                                   LI2F0230
C                         ON OUTPUT, IDGT CONTAINS THE APPROXIMATE      LI2F0240
C                           NUMBER OF DIGITS IN THE ANSWER WHICH        LI2F0250
C                           WERE UNCHANGED AFTER IMPROVEMENT.           LI2F0260
C                WKAREA - WORK AREA OF DIMENSION GREATER THAN OR EQUAL  LI2F0270
C                           TO N**2+3N.                                 LI2F0280
C                IER    - ERROR PARAMETER                               LI2F0290
C                         TERMINAL ERROR = 128+N.                       LI2F0300
C                           N = 1 INDICATES THAT THE MATRIX IS          LI2F0310
C                             ALGORITHMICALLY SINGULAR. (SEE THE CHAPTERLI2F0320
C                             L PRELUDE).                               LI2F0330
C                           N = 3 INDICATES THAT THE MATRIX IS TOO      LI2F0340
C                             ILL-CONDITIONED FOR ITERATIVE IMPROVEMENT LI2F0350
C                             TO BE EFFECTIVE.                          LI2F0360
C                         WARNING ERROR = 32+N.                         LI2F0370
C                           N = 2 INDICATES THAT THE ACCURACY TEST      LI2F0380
C                             FAILED.                                   LI2F0390
C                             THE COMPUTED SOLUTION MAY BE IN ERROR BY  LI2F0400
C                             MORE THAN CAN BE ACCOUNTED FOR BY THE     LI2F0410
C                             UNCERTAINTY OF THE DATA.                  LI2F0420
C                             THIS WARNING CAN BE PRODUCED ONLY IF IDGT LI2F0430
C                             IS GREATER THAN 0 ON INPUT.               LI2F0440
C                             SEE CHAPTER L PRELUDE FOR FURTHER         LI2F0450
C                             DISCUSSION.                               LI2F0460
C   PRECISION           - SINGLE                                        LI2F0470
C   REQD. IMSL ROUTINES - LEQT2F,LUDATF,LUELMF,LUREFF,UERTST            LI2F0480
C   LANGUAGE            - FORTRAN                                       LI2F0490
C-----------------------------------------------------------------------LI2F0500
C   LATEST REVISION     - AUGUST 15, 1973                               LI2F0510
C                                                                       LI2F0520
      DIMENSION          A(IA,1),AINV(IA,1),WKAREA(1)                   LI2F0530
      DATA               ONE/1.0/,ZERO/0.0/                             LI2F0540
C-------------------------------INITIALIZE IER--------------------------LI2F0550
```

```
      IER=0                                                             LI2F0560
C                                  SET AINV TO THE N X N                LI2F0570
C                                  IDENTITY MATRIX                      LI2F0580
      DO 10 I = 1,N                                                     LI2F0590
         DO 5 J = 1,N                                                   LI2F0600
            AINV(I,J) = ZERO                                            LI2F0610
    5    CONTINUE                                                       LI2F0620
         AINV(I,I) = ONE                                                LI2F0630
   10 CONTINUE                                                          LI2F0640
C                                  COMPUTE THE INVERSE OF A             LI2F0650
      CALL LEQT2F (A,N,N,IA,AINV,IDGT,WKAREA,IER)                       LI2F0660
      IF (IER.EQ.0) GO TO 9005                                          LI2F0670
 9000 CONTINUE                                                          LI2F0680
      CALL UERTST (IER,6HLINV2F)                                        LI2F0690
 9005 RETURN                                                            LI2F0700
      END                                                               LI2F0710
      SUBROUTINE UERTST (IER,NAME)                                      UERT0010
C                                                                       UERT0020
C-UERTST----------------LIBRARY 3---------------------------------------UERT0030
C                                                                       UERT0040
C   FUNCTION            - ERROR MESSAGE GENERATION                      UERT0050
C   USAGE               - CALL UERTST(IER,NAME)                         UERT0060
C   PARAMETERS   IER    - ERROR PARAMETER. TYPE + N  WHERE              UERT0070
C                           TYPE= 128 IMPLIES TERMINAL ERROR            UERT0080
C                                  64 IMPLIES WARNING WITH FIX          UERT0090
C                                  32 IMPLIES WARNING                   UERT0100
C                           N    = ERROR CODE RELEVANT TO CALLING ROUTINEUERT0110
C                NAME   - INPUT SCALAR CONTAINING THE NAME OF THE       UERT0120
C                           CALLING ROUTINE AS A 6-CHARACTER LITERAL    UERT0130
C                           STRING.                                     UERT0140
C   LANGUAGE            - FORTRAN                                       UERT0150
C-----------------------------------------------------------------------UERT0160
C   LATEST REVISION     - AUGUST 1, 1973                                UERT0170
C                                                                       UERT0180
      DIMENSION          ITYP(2,4),IBIT(4)                              UERT0190
      INTEGER            WARN,WARF,TERM,PRINTR                          UERT0200
      EQUIVALENCE        (IBIT(1),WARN),(IBIT(2),WARF),(IBIT(3),TERM)   UERT0210
      DATA      ITYP      /10HWARNING   ,10H          ,                 UERT0220
     *                    10HWARNING(WI,10HTH FIX)    ,                 UERT0230
     *                    10HTERMINAL  ,10H           ,                 UERT0240
     *                    10HNON-DEFINE,10HD          /,                UERT0250
     *          IBIT      / 32,64,128,0/                                UERT0260
      DATA               PRINTR/6LOUTPUT/                               UERT0270
      IER2=IER                                                          UERT0280
      IF (IER2 .GE. WARN) GO TO 5                                       UERT0290
C                                  NON-DEFINED                          UERT0300
      IER1=4                                                            UERT0310
      GO TO 20                                                          UERT0320
    5 IF (IER2 .LT. TERM) GO TO 10                                      UERT0330
C                                  TERMINAL                             UERT0340
      IER1=3                                                            UERT0350
      GO TO 20                                                          UERT0360
   10 IF (IER2 .LT. WARF) GO TO 15                                      UERT0370
C                                  WARNING(WITH FIX)                    UERT0380
      IER1=2                                                            UERT0390
      GO TO 20                                                          UERT0400
C                                  WARNING                              UERT0410
   15 IER1=1                                                            UERT0420
C                                  EXTRACT *N*                          UERT0430
   20 IER2=IER2-IBIT(IER1)                                              UERT0440
C                                  PRINT ERROR MESSAGE                  UERT0450
      WRITE (PRINTR,25) (ITYP(I,IER1),I=1,2),NAME,IER2,IER              UERT0460
   25 FORMAT(26H *** I M S L(UERTST) ***  ,2A10,4X,A6,4X,I2,            UERT0470
     1   8H (IER = ,I3,1H))                                             UERT0480
      RETURN                                                            UERT0490
      END                                                               UERT0500
```

```
      SUBROUTINE LUELMF (A,B,IPVT,N,IA,X)
C
C-LUELMF--------S-------LIBRARY 3----------------------------------------
C
C   FUNCTION            - ELIMINATION PART OF SOLUTION OF AX=B -
C                           FULL STORAGE MODE
C   USAGE               - CALL LUELMF (A,B,IPVT,N,IA,X)
C   PARAMETERS   A      - THE RESULT, LU, COMPUTED IN THE SUBROUTINE
C                           *LUDATF*, WHERE L IS A LOWER TRIANGULAR
C                           MATRIX WITH ONES ON THE MAIN DIAGONAL. U IS
C                           UPPER TRIANGULAR. L AND U ARE STORED AS A
C                           SINGLE MATRIX A, AND THE UNIT DIAGONAL OF
C                           L IS NOT STORED
C                B      - B IS A VECTOR OF LENGTH N ON THE RIGHT HAND
C                           SIDE OF THE EQUATION AX=B
C                IPVT   - THE PERMUTATION MATRIX RETURNED FROM THE
C                           SUBROUTINE *LUDATF*, STORED AS AN N LENGTH
C                           VECTOR
C                N      - ORDER OF A AND NUMBER OF ROWS IN B
C                IA     - NUMBER OF ROWS IN THE DIMENSION STATEMENT
C                           FOR A IN THE CALLING PROGRAM.
C                X      - THE RESULT X
C   PRECISION           - SINGLE
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - APRIL 11,1975
C
      DIMENSION          A(IA,1),B(1),IPVT(1),X(1)
C                                  SOLVE LY = B FOR Y
      DO 5 I=1,N
    5 X(I) = B(I)
      IW = 0
      DO 20 I=1,N
         IP = IPVT(I)
         SUM = X(IP)
         X(IP) = X(I)
         IF (IW .EQ. 0) GO TO 15
         IM1 = I-1
         DO 10 J=IW,IM1
            SUM = SUM-A(I,J)*X(J)
   10    CONTINUE
         GO TO 20
   15    IF (SUM .NE. 0.) IW = I
   20 X(I) = SUM
C                                  SOLVE UX = Y FOR X
      DO 30 IB=1,N
         I = N+1-IB
         IP1 = I+1
         SUM = X(I)
         IF (IP1 .GT. N) GO TO 30
         DO 25 J=IP1,N
            SUM = SUM-A(I,J)*X(J)
   25   CONTINUE
   30 X(I) = SUM/A(I,I)
      RETURN
      END
      SUBROUTINE LEQT2F (A,M,N,IA,B,IDGT,WKAREA,IER)
C
C-LEQT2F--------S-------LIBRARY 3----------------------------------------
C
C   FUNCTION            - LINEAR EQUATION SOLUTION - FULL STORAGE
C                           MODE - HIGH ACCURACY SOLUTION.
C   USAGE               - CALL LEQT2F (A,M,N,IA,B,IDGT,WKAREA,IER)
C   PARAMETERS   A      - INPUT MATRIX OF DIMENSION N BY N CONTAINING
C                           THE COEFFICIENT MATRIX OF THE EQUATION
C                           AX = B.
```

```
C                 M       - NUMBER OF RIGHT-HAND SIDES.(INPUT)
C                 N       - ORDER OF A AND NUMBER OF ROWS IN B.(INPUT)
C                 IA      - NUMBER OF ROWS IN THE DIMENSION STATEMENT
C                             FOR A AND B IN THE CALLING PROGRAM.(INPUT)
C                 B       - INPUT MATRIX OF DIMENSION N BY M CONTAINING
C                             THE RIGHT-HAND SIDES OF THE EQUATION AX = B.
C                           ON OUTPUT, THE N BY M MATRIX OF SOLUTIONS
C                             REPLACES B.
C                 IDGT    - INPUT OPTION.
C                           IF IDGT IS GREATER THAN 0, THE ELEMENTS OF
C                             A AND B ARE ASSUMED TO BE CORRECT TO IDGT
C                             DECIMAL DIGITS AND THE ROUTINE PERFORMS
C                             AN ACCURACY TEST.
C                           IF IDGT EQUALS 0, THE ACCURACY TEST IS
C                             BYPASSED.
C                           ON OUTPUT, IDGT CONTAINS THE APPROXIMATE
C                             NUMBER OF DIGITS IN THE ANSWER WHICH
C                             WERE UNCHANGED AFTER IMPROVEMENT.
C                 WKAREA  - WORK AREA OF DIMENSION GREATER THAN OR EQUAL
C                             TO N**2+3N.
C                 IER     - ERROR PARAMETER
C                           WARNING ERROR = 32+N.
C                             N = 2 INDICATES THAT THE ACCURACY TEST
C                                   FAILED.
C                                   THE COMPUTED SOLUTION MAY BE IN ERROR
C                                   BY MORE THAN CAN BE ACCOUNTED FOR BY
C                                   THE UNCERTAINTY OF THE DATA.
C                                   THIS WARNING CAN BE PRODUCED ONLY IF
C                                   IDGT IS GREATER THAN 0 ON INPUT.
C                                   SEE CHAPTER L PRELUDE FOR FURTHER
C                                   DISCUSSION.
C                           TERMINAL ERROR = 128+N.
C                             N = 1 INDICATES THAT THE MATRIX IS
C                               ALGORITHMICALLY SINGULAR. (SEE THE
C                               CHAPTER L PRELUDE).
C                             N = 3 INDICATES THAT THE MATRIX IS TOO
C                               ILL-CONDITIONED FOR ITERATIVE IMPROVEMENT
C                               TO BE EFFECTIVE.
C   PRECISION           - SINGLE
C   REQD. IMSL ROUTINES - LUDATF,LUELMF,LUREFF,UERTST
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - AUGUST 15,1973
C
      DIMENSION          A(IA,1),B(IA,1),WKAREA(1)
C                                  INITIALIZE IER
      IER=0
      JER=0
      J = N*N+1
      K = J+N
      MM = K+N
      KK = 0
      MM1 = MM-1
      JJ=1
      DO 5 L=1,N
         DO 5 I=1,N
            WKAREA(JJ)=A(I,L)
            JJ=JJ+1
    5 CONTINUE
C                                  DECOMPOSE A
      CALL LUDATF (WKAREA(1),A,N,N,IDGT,D1,D2,WKAREA(J),WKAREA(K),
     1   WA,IER)
      IF (IER.GT.128) GO TO 25
      IF (IDGT .EQ. 0 .OR. IER .NE. 0) KK = 1
      DO 15 I = 1,M
C                                  PERFORMS THE ELIMINATION PART OF
```

```
C                                   AX = B                              LE2F0770
         CALL LUELMF (A,B(1,I),WKAREA(J),N,N,WKAREA(MM))                LE2F0780
C                                   REFINEMENT OF SOLUTION TO AX = B    LE2F0790
         IF (KK .NE. 0)                                                 LE2F0800
     *   CALL LUREFF (WKAREA(1),B(1,I),A,WKAREA(J),N,N,WKAREA(MM),IDGT, LE2F0810
     *               WKAREA(K),WKAREA(K),JER)                           LE2F0820
         DO 10 II=1,N                                                   LE2F0830
            B(II,I) = WKAREA(MM1+II)                                    LE2F0840
   10    CONTINUE                                                       LE2F0850
         IF (JER.NE.0) GO TO 20                                         LE2F0860
   15 CONTINUE                                                          LE2F0870
      GO TO 25                                                          LE2F0880
   20 IER = 131                                                         LE2F0890
   25 JJ=1                                                              LE2F0900
      DO 30 J=1,N                                                       LE2F0910
         DO 30 I=1,N                                                    LE2F0920
            A(I,J)=WKAREA(JJ)                                           LE2F0930
            JJ=JJ+1                                                     LE2F0940
   30 CONTINUE                                                          LE2F0950
      IF (IER .EQ. 0) GO TO 9005                                        LE2F0960
 9000 CONTINUE                                                          LE2F0970
      CALL UERTST (IER,6HLEQT2F)                                        LE2F0980
 9005 RETURN                                                            LE2F0990
      END                                                               LE2F1000
      SUBROUTINE LUREFF (A,B,UL,IPVT,N,IA,X,IDGT,RES,DX,IER)            LURF0010
C                                                                       LURF0020
C-LUREFF--------S-------LIBRARY 3---------------------------------------LURF0030
C                                                                       LURF0040
C   FUNCTION            - REFINEMENT OF SOLUTION TO LINEAR EQUATIONS -  LURF0050
C                           FULL STORAGE MODE                           LURF0060
C   USAGE               - CALL LUREFF (A,B,UL,IPVT,N,IA,X,IDGT,RES,DX,  LURF0070
C                           IER)                                        LURF0080
C   PARAMETERS   A      - THE COEFFICIENT MATRIX, AX=B, WHERE A         LURF0090
C                           IS N X N                                    LURF0100
C                B      - THE RIGHT HAND SIDE, A VECTOR OF SIZE N       LURF0110
C                UL     - A GIVEN N X N MATRIX, UL IS THE LU            LURF0120
C                           DECOMPOSITION OF A AS SUPPLIED BY           LURF0130
C                           IMSL ROUTINE *LUDATF*                       LURF0140
C                IPVT   - A GIVEN VECTOR OF PIVOT INDICES OF SIZE N     LURF0150
C                           AS SUPPLIED BY IMSL ROUTINE *LUDATF*        LURF0160
C                N      - ORDER OF A AND UL, AND ALSO IS THE LENGTH OF  LURF0170
C                           B, IPVT, X, RES, AND DX                     LURF0180
C                IA     - ROW DIMENSION OF A AND UL IN THE CALLING      LURF0190
C                           PROGRAM                                     LURF0200
C                X      - A GIVEN VECTOR OF SIZE N, X IS AN ESTIMATE    LURF0210
C                           TO THE SOLUTION OF AX=B. THE IMPROVED RESULTLURF0220
C                           OVERWRITES THE INPUT VECTOR X               LURF0230
C                IDGT   - APPROXIMATE NUMBER OF DIGITS IN THE ANSWER    LURF0240
C                           WHICH WERE UNCHANGED AFTER IMPROVEMENT      LURF0250
C                RES    - THE RESIDUAL VECTOR OF SIZE N, USED AS A WORK LURF0260
C                           VECTOR                                      LURF0270
C                DX     - A WORK VECTOR OF SIZE N                       LURF0280
C                IER    - ERROR PARAMETER                               LURF0290
C                           TERMINAL ERROR = 128 + N                    LURF0300
C                             N = 1, INDICATES ITERATIVE IMPROVEMENT    LURF0310
C                                    FAILED. MATRIX IS TOO ILL          LURF0320
C                                    CONDITIONED.                       LURF0330
C   PRECISION           - SINGLE                                        LURF0340
C   REQD. IMSL ROUTINES - LUELMF,UERTST                                 LURF0350
C   LANGUAGE            - FORTRAN                                       LURF0360
C-----------------------------------------------------------------------LURF0370
C   LATEST REVISION     - AUGUST 15, 1973                               LURF0380
C                                                                       LURF0390
      DIMENSION          A(IA,1),UL(IA,1),B(1),X(1),RES(1),DX(1),IPVT(1)LURF0400
      DOUBLE PRECISION   SUM                                            LURF0410
      DATA               ITMAX/50/,ZERO/0.0/                            LURF0420
```

Continuation Example of Output for Program COMBI

```
SUMMARY OF STATISTICAL VALUES:(FIRST   75 POINTS OF DATA CONSIDERED)
METHOD:                               BOX JENKIN     SMOOTHING

(1) SUM OF ERRORS:                    -3.3272        -4.7496

(2) MEANS:                             -.0475         -.0660

(3) SUM(ERROR-MEAN):                   -.0475         -.1979

(4) VARIANCE:                           .0312          .0364

(5) STANDARD DEVIATION:                 .1766          .1907

(6) MEAN SQUARE ERROR:                  .0331          .0413

NUMBER OF ERRORS USED IN COMBINATION:    68

CORRELATION COEFFICIENTS:
                    BOX JENK         SMOOTHIN
BOX JENK              1.0000
SMOOTHIN               .9505           1.0000

COVARIANCE MATRIX:
                    BOX JENK         SMOOTHIN
BOX JENK               .0312            .0320
SMOOTHIN               .0320            .0364

INVERSE OF COVARIANCE MATRIX:
                    BOX JENK         SMOOTHIN
BOX JENK            332.1134        -292.3080
SMOOTHIN           -292.3080         284.7638

WEIGHT VECTOR FOR COMBINED FORECASTS
     .1234E+01
    -.2338E+00

COMBINATION OF    2 METHODS BOX JENKIN  SMOOTHING
                                                       ERROR VARIANCE=          .0310
                                                       % IMPROVEMENT OVER MIN. VARIANCE:    .62
```

```
      IER=0                                                             LURF0430
      XNORM = 0.0                                                       LURF0440
      DO 10 I=1,N                                                       LURF0450
         XNORM = AMAX1(XNORM,ABS(X(I)))                                 LURF0460
   10 CONTINUE                                                          LURF0470
      IF (XNORM .NE. 0.) GO TO 20                                       LURF0480
      IDGT = 50                                                         LURF0490
      GO TO 9005                                                        LURF0500
   20 DO 45 ITER=1,ITMAX                                                LURF0510
         DO 30 I=1,N                                                    LURF0520
            SUM = DBLE(B(I))                                            LURF0530
            DO 25 J=1,N                                                 LURF0540
               SUM = SUM - DBLE(A(I,J)) * DBLE(X(J))                    LURF0550
   25       CONTINUE                                                    LURF0560
            RES(I) = SUM                                                LURF0570
   30    CONTINUE                                                       LURF0580
         CALL LUELMF (UL,RES,IPVT,N,IA,DX)                              LURF0590
         DXNORM = 0.0                                                   LURF0600
         DO 35 I=1,N                                                    LURF0610
            X(I) = X(I) + DX(I)                                         LURF0620
            DXNORM = AMAX1(DXNORM,ABS(DX(I)))                           LURF0630
   35    CONTINUE                                                       LURF0640
         IF (ITER .NE. 1) GO TO 40                                      LURF0650
         IDGT = 50                                                      LURF0660
         IF (DXNORM .NE. ZERO) IDGT = -ALOG10(DXNORM/XNORM)             LURF0670
   40    IF (XNORM+DXNORM .EQ. XNORM) GO TO 9005                        LURF0680
   45 CONTINUE                                                          LURF0690
C                                  ITERATION DID NOT CONVERGE           LURF0700
      IER = 129                                                         LURF0710
 9000 CONTINUE                                                          LURF0720
      CALL UERTST(IER,6HLUREFF)                                         LURF0730
 9005 RETURN                                                            LURF0740
      END                                                               LURF0750
      SUBROUTINE LUDATF (A,LU,N,IA,IDGT,D1,D2,IPVT,EQUIL,WA,IER)        LUDA0010
C                                                                       LUDA0020
C-LUDATF--------S-------LIBRARY 3---------------------------------------LUDA0030
C                                                                       LUDA0040
C   FUNCTION            - L-U DECOMPOSITION BY THE CROUT ALGORITHM      LUDA0050
C                           WITH OPTIONAL ACCURACY TEST.                LUDA0060
C   USAGE               - CALL LUDATF(A,LU,N,IA,IDGT,D1,D2,IPVT,        LUDA0070
C                                EQUIL,WA,IER)                          LUDA0080
C   PARAMETERS   A      - INPUT MATRIX OF DIMENSION N BY N CONTAINING   LUDA0090
C                           THE MATRIX TO BE DECOMPOSED                 LUDA0100
C                LU     - REAL OUTPUT MATRIX OF DIMENSION N BY N        LUDA0110
C                           CONTAINING THE L-U DECOMPOSITION OF A       LUDA0120
C                           ROWWISE PERMUTATION OF THE INPUT MATRIX.    LUDA0130
C                           FOR A DESCRIPTION OF THE FORMAT OF LU, SEE  LUDA0140
C                           EXAMPLE.                                    LUDA0150
C                N      - INPUT SCALAR CONTAINING THE ORDER OF THE      LUDA0160
C                           MATRIX A.                                   LUDA0170
C                IA     - INPUT SCALAR CONTAINING THE ROW DIMENSION OF  LUDA0180
C                           MATRICES A AND LU IN THE CALLING PROGRAM.   LUDA0190
C                IDGT   - INPUT OPTION.                                 LUDA0200
C                           IF IDGT IS GREATER THAN ZERO, THE NON-ZERO  LUDA0210
C                           ELEMENTS OF A ARE ASSUMED TO BE CORRECT TO  LUDA0220
C                           IDGT DECIMAL PLACES.  LUDATF PERFORMS AN    LUDA0230
C                           ACCURACY TEST TO DETERMINE IF THE COMPUTED  LUDA0240
C                           DECOMPOSITION IS THE EXACT DECOMPOSITION    LUDA0250
C                           OF A MATRIX WHICH DIFFERS FROM THE GIVEN ONELUDA0260
C                           BY LESS THAN ITS UNCERTAINTY.               LUDA0270
C                         IF IDGT IS EQUAL TO ZERO, THE ACCURACY TEST ISLUDA0280
C                           BYPASSED.                                   LUDA0290
C                D1     - OUTPUT SCALAR CONTAINING ONE OF THE TWO       LUDA0300
C                           COMPONENTS OF THE DETERMINANT. SEE          LUDA0310
C                           DESCRIPTION OF PARAMETER D2, BELOW.         LUDA0320
C                D2     - OUTPUT SCALAR CONTAINING ONE OF THE           LUDA0330
```

```
C                             TWO COMPONENTS OF THE DETERMINANT. THE
C                             DETERMINANT MAY BE EVALUATED AS (D1)(2**D2)
C               IPVT    - OUTPUT VECTOR OF LENGTH N CONTAINING THE
C                             PERMUTATION INDICES. SEE DOCUMENT
C                             (ALGORITHM).
C               EQUIL   - OUTPUT VECTOR OF LENGTH N CONTAINING
C                             RECIPROCALS OF THE ABSOLUTE VALUES OF
C                             THE LARGEST (IN ABSOLUTE VALUE) ELEMENT
C                             IN EACH ROW.
C               WA      - ACCURACY TEST PARAMETER, OUTPUT ONLY IF
C                             IDGT IS GREATER THAN ZERO.
C                             SEE ELEMENT DOCUMENTATION FOR DETAILS.
C               IER     - ERROR PARAMETER
C                           TERMINAL ERROR=128+N
C                             N = 1 INDICATES THAT MATRIX A IS
C                                   ALGORITHMICALLY SINGULAR. (SEE THE
C                                   CHAPTER L PRELUDE).
C                           WARNING ERROR=32+N
C                             N = 2 INDICATES THAT THE ACCURACY TEST
C                                   FAILED.
C                                   THE COMPUTED SOLUTION MAY BE IN ERROR
C                                   BY MORE THAN CAN BE ACCOUNTED FOR BY
C                                   THE UNCERTAINTY OF THE DATA.
C                                   THIS WARNING CAN BE PRODUCED ONLY IF
C                                   IDGT IS GREATER THAN 0 ON INPUT.
C                                   SEE CHAPTER L PRELUDE FOR FURTHER
C                                   DISCUSSION.
C   PRECISION           - SINGLE
C   REQD. IMSL ROUTINES - UERTST
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - AUGUST 15, 1973
C
      DIMENSION          A(IA,1),LU(IA,1),IPVT(1),EQUIL(1)
      REAL               LU
      DATA               ZERO,ONE,FOUR,SIXTN,SIXTH/0.0,1.,4.,16.,.0625/
C                                  INITIALIZATION
      IER = 0
      RN = N
      WREL = ZERO
      D1 = ONE
      D2 = ZERO
      BIGA = ZERO
      DO 10 I=1,N
         BIG = ZERO
         DO 5 J=1,N
            P = A(I,J)
            LU(I,J) = P
            P = ABS(P)
            IF (P .GT. BIG) BIG = P
    5    CONTINUE
         IF (BIG .GT. BIGA) BIGA = BIG
         IF (BIG .EQ. ZERO) GO TO 110
         EQUIL(I) = ONE/BIG
   10 CONTINUE
      DO 105 J=1,N
         JM1 = J-1
         IF (JM1 .LT. 1) GO TO 40
C                                  COMPUTE U(I,J), I=1,...,J-1
         DO 35 I=1,JM1
            SUM = LU(I,J)
            IM1 = I-1
            IF (IDGT .EQ. 0) GO TO 25
C                                  WITH ACCURACY TEST
            AI = ABS(SUM)
            WI = ZERO
```

```
            IF (IM1 .LT. 1) GO TO 20                                    LUDA1000
            DO 15 K=1,IM1                                               LUDA1010
               T = LU(I,K)*LU(K,J)                                      LUDA1020
               SUM = SUM-T                                              LUDA1030
               WI = WI+ABS(T)                                           LUDA1040
   15       CONTINUE                                                    LUDA1050
            LU(I,J) = SUM                                               LUDA1060
   20       WI = WI+ABS(SUM)                                            LUDA1070
            IF (AI .EQ. ZERO) AI = BIGA                                 LUDA1080
            TEST = WI/AI                                                LUDA1090
            IF (TEST .GT. WREL) WREL = TEST                             LUDA1100
            GO TO 35                                                    LUDA1110
C                                  WITHOUT ACCURACY                     LUDA1120
   25       IF (IM1 .LT. 1) GO TO 35                                    LUDA1130
            DO 30 K=1,IM1                                               LUDA1140
               SUM = SUM-LU(I,K)*LU(K,J)                                LUDA1150
   30       CONTINUE                                                    LUDA1160
            LU(I,J) = SUM                                               LUDA1170
   35    CONTINUE                                                       LUDA1180
   40    P = ZERO                                                       LUDA1190
C                                  COMPUTE U(J,J) AND L(I,J), I=J+1,...,LUDA1200
         DO 70 I=J,N                                                    LUDA1210
            SUM = LU(I,J)                                               LUDA1220
            IF (IDGT .EQ. 0) GO TO 55                                   LUDA1230
C                                  WITH ACCURACY TEST                   LUDA1240
            AI = ABS(SUM)                                               LUDA1250
            WI = ZERO                                                   LUDA1260
            IF (JM1 .LT. 1) GO TO 50                                    LUDA1270
            DO 45 K=1,JM1                                               LUDA1280
               T = LU(I,K)*LU(K,J)                                      LUDA1290
               SUM = SUM-T                                              LUDA1300
               WI = WI+ABS(T)                                           LUDA1310
   45       CONTINUE                                                    LUDA1320
            LU(I,J) = SUM                                               LUDA1330
   50       WI = WI+ABS(SUM)                                            LUDA1340
            IF (AI .EQ. ZERO) AI = BIGA                                 LUDA1350
            TEST = WI/AI                                                LUDA1360
            IF (TEST .GT. WREL) WREL = TEST                             LUDA1370
            GO TO 65                                                    LUDA1380
C                                  WITHOUT ACCURACY TEST                LUDA1390
   55       IF (JM1 .LT. 1) GO TO 65                                    LUDA1400
            DO 60 K=1,JM1                                               LUDA1410
               SUM = SUM-LU(I,K)*LU(K,J)                                LUDA1420
   60       CONTINUE                                                    LUDA1430
            LU(I,J) = SUM                                               LUDA1440
   65       Q = EQUIL(I)*ABS(SUM)                                       LUDA1450
            IF (P .GE. Q) GO TO 70                                      LUDA1460
            P = Q                                                       LUDA1470
            IMAX = I                                                    LUDA1480
   70    CONTINUE                                                       LUDA1490
C                                  TEST FOR ALGORITHMIC SINGULARITY     LUDA1500
         IF (RN+P .EQ. RN) GO TO 110                                    LUDA1510
         IF (J .EQ. IMAX) GO TO 80                                      LUDA1520
C                                  INTERCHANGE ROWS J AND IMAX          LUDA1530
         D1 = -D1                                                       LUDA1540
         DO 75 K=1,N                                                    LUDA1550
            P = LU(IMAX,K)                                              LUDA1560
            LU(IMAX,K) = LU(J,K)                                        LUDA1570
            LU(J,K) = P                                                 LUDA1580
   75    CONTINUE                                                       LUDA1590
         EQUIL(IMAX) = EQUIL(J)                                         LUDA1600
   80    IPVT(J) = IMAX                                                 LUDA1610
         D1 = D1*LU(J,J)                                                LUDA1620
   85    IF (ABS(D1) .LE. ONE) GO TO 90                                 LUDA1630
         D1 = D1*SIXTH                                                  LUDA1640
         D2 = D2+FOUR                                                   LUDA1650
```

```
      GO TO 85
   90 IF (ABS(D1) .GE. SIXTH) GO TO 95
      D1 = D1*SIXTN
      D2 = D2-FOUR
      GO TO 90
   95 CONTINUE
      JP1 = J+1
      IF (JP1 .GT. N) GO TO 105
C                                  DIVIDE BY PIVOT ELEMENT U(J,J)
      P = LU(J,J)
      DO 100 I=JP1,N
         LU(I,J) = LU(I,J)/P
  100 CONTINUE
  105 CONTINUE
C                                  PERFORM ACCURACY TEST
      IF (IDGT .EQ. 0) GO TO 9005
      P = 3*N+3
      WA = P*WREL
      IF (WA+10.0**(-IDGT) .NE. WA) GO TO 9005
      IER = 34
      GO TO 9000
C                                  ALGORITHMIC SINGULARITY
  110 IER = 129
      D1 = ZERO
      D2 = ZERO
 9000 CONTINUE
C                                  PRINT ERROR
      CALL UERTST(IER,6HLUDATF)
 9005 RETURN
      END
      SUBROUTINE PLOT(XOBS,XFOR,NOBS,NFOR,ISCALE,OPTION,JDIV,IDIV)
      DIMENSION XOBS(500),XFOR(1000),GRAPH(100),Y(11)
      DATA OBS,FOR,BLANK,DOT,PLUS,DASH/1HO,1HF,1H ,1H.,1H+,1H-/
C     ******************************************************
C     * XOBS IS A VECTOR OF ACTUAL OBSERVATIONS.           *
C     * XFOR IS THE VECTOR OF FORECASTING.                 *
C     * NOBS IS # OF PERIODS OBSERVED>                     *
C     * NFOR IS # OF PERIODS FORCASTED.                    *
C     * ISCALE SETS THE WIDTH OF THE PLOT. 50 OR 100 COLMNS*
C     * FOR PARTITIONING OF THE GRAPH:                     *
C     *      OPTION=0. WILL SUPPRESS PARTITIONING.         *
C     *      OPTION=1. WILL PROVIDE PARTITIONING,          *
C     *                EVERY 10TH COL. AND 12TH ROW.       *
C     *      OPTION=2. SET YOUR OWN PARTITIONS.            *
C     *                                                    *
C     * WHEN OPTION 2 IS CHOSEN:                           *
C     *      JDIV IS THE HORIZONTAL PARTITION.             *
C     *      IDIV IS THE VERTICAL PARTITION.               *
C     * WHEN OBSERVED & FORCASTED ARE TO BE PLOTTED        *
C     * TOGETHER, ONLY 'F' WILL BE PRINTED.                *
C     *                                                    *
C     * THIS PLOTTER WAS CONTRIBUTED TO THE BENEFIT OF     *
C     * ALL BY GADI NAAMAN & ROBERT ALEXANDER.             *
C     ******************************************************
      IF(OPTION.EQ.2.) GO TO 3
      IDIV=12
      JDIV=10
    3 ITOP=NFOR
      RMAX=XOBS(1)
      RMIN=XOBS(1)
      IF(NOBS.GT.NFOR) ITOP=NOBS
      DO 1 I=1,ITOP
      IF(I.GT.NOBS) GO TO 2
      IF(XOBS(I).GE.RMAX) RMAX=XOBS(I)
      IF(XOBS(I).LE.RMIN) RMIN=XOBS(I)
    2 IF(I.GT.NFOR) GO TO 1
```

```
      IF(XFOR(I).GE.RMAX) RMAX=XFOR(I)
      IF(XFOR(I).LE.RMIN) RMIN=XFOR(I)
    1 CONTINUE
      SCALE=FLOAT(ISCALE)
      DIV=(RMAX-RMIN)/SCALE
      PRINT*,'RMIN=',RMIN,' RMAX=',RMAX,' OPTION=',OPTION,
     +'JDIV=',JDIV
          ENDFILE 6
      INDX=0
      JSCALE=ISCALE+1
      DO 10 K=1,JSCALE,10
      INDX=INDX+1
      Y(INDX)=RMIN+DIV*(K-1)
   10 CONTINUE
      IF(ISCALE.EQ.50) WRITE(6,15) (Y(I),I=1,INDX)
   15 FORMAT(1H1,15X,*PLOT OF OBSERVED & FORCASTED TIME SERIES*
     +/16X,40(*-*)////1HQ/
     +1X,6(F8.2,2X)/6X,5(*.........+*)/58X,*OBSERV*,3X,*FORCST*/1HR/)
      IF(ISCALE.EQ.100) WRITE(6,20) (Y(I),I=1,INDX)
  20  FORMAT(1H1,45X,'PLOT OF OBSERVED & FORECASTED TIME SERIES'
     +/46X,40('-')////1HQ/
     +1X,11(F8.2,2X)/6X,10(*.........+*)/108X,*OBSERV*,3X,*FORCST*/1HR/)
      DO 8 I=1,ITOP
      IXOBS=DOT
      IXFOR=DOT
      IF(I.LE.NOBS) IXOBS=(XOBS(I)-RMIN)/DIV+.05
      IF(I.LE.NFOR) IXFOR=(XFOR(I)-RMIN)/DIV+.5
      IPART=(I/IDIV)*IDIV
      DO 9 J=1,JSCALE
      JPART=(J/JDIV)*JDIV
      GRAPH(J)=BLANK
      IF(OPTION.EQ.0) GO TO 9
      IF(IPART.EQ.I.AND.((J/2)*2).EQ.J) GRAPH(J)=DASH
      IF(JPART.EQ.J.AND.((I/2)*2).EQ.I) GRAPH(J)=DOT
      IF(IPART.EQ.I.AND.JPART.EQ.J) GRAPH(J)=PLUS
    9 CONTINUE
      IF(IXOBS.NE.DOT.AND.IXOBS.NE.0) GRAPH(IXOBS)=OBS
      IF(IXFOR.NE.DOT.AND.IXFOR.NE.0) GRAPH(IXFOR)=FOR
      ICHAR = I
      IF(I.LE.NOBS.AND.I.LE.NFOR)
     +WRITE(6,17)ICHAR,ISCALE,(GRAPH(J),J=1,ISCALE),XOBS(I),XFOR(I)
      IF(I.GT.NOBS)
     +WRITE(6,17) ICHAR,ISCALE,(GRAPH(J),J=1,ISCALE),XFOR(I),XFOR(I)
      IF(I.GT.NFOR) WRITE(6,17) ICHAR,ISCALE,
     +(GRAPH(J),J=1,ISCALE),XOBS(I)
   17 FORMAT(2X,I3,*+*,=A1,F8.2,*,*,F8.2)
    8 CONTINUE
   16 CONTINUE
      RETURN
      END
```

Example of output for computer program COMBI

THESIS: A STATISTICAL APPROACH TO THE COMBINATION OF FORECASTS

GENOVEVA CRUZ FALL 1977

A LINEAR COMBINATION OF FORECASTS CAN GIVE A SMALLER ERROR VARIANCE THAN ANY OF THE INDIVIDUAL FORECASTING METHODS...

DELTA AIRLINES

DATA LISTING:

PERIOD	DATA	METHOD 1 ADAPTIVE	ERRORS	METHOD 2 BOX JENKIN	ERRORS	METHOD 3 WINTERS	ERRORS	METHOD 4 REGRESSION	ERRORS	METHOD 5 SMOOTHING	ERRORS
1	14.1000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	13.9800	.1222	0.0000	0.0000
2	14.0000	0.0000	0.0000	0.0000	0.0000	14.0800	-.0820	13.9500	.0455	0.0000	0.0000
3	14.0000	0.0000	0.0000	14.0000	-.0015	14.0300	-.0255	13.9300	.0688	0.0000	0.0000
4	14.0000	0.0000	0.0800	14.0000	-.0009	13.9200	.0760	13.9100	.0921	0.0000	0.0000
5	14.1000	13.9500	.1493	14.0000	.0995	14.0000	.1022	13.8800	.2154	0.0000	0.0000
6	13.7000	14.0100	-.3190	14.0400	-.3399	13.9800	-.2810	13.8600	-.1613	0.0000	0.0000
7	13.7000	13.8100	-.1144	13.9000	-.2050	13.7500	-.0504	13.8400	-.1380	0.0000	0.0000
8	13.7000	13.7000	.0046	13.8200	-.1203	13.7600	-.0605	13.8100	-.1147	13.9000	-.1983
9	13.7000	13.6200	.0777	13.7700	-.0707	13.7600	-.0553	13.7900	-.0913	13.8400	-.1388
10	13.6000	13.5500	.0478	13.7400	-.1415	13.5400	.0622	13.7700	-.1680	13.8000	-.1972
11	13.6000	13.5500	.0526	13.6800	-.0846	13.6200	-.0237	13.7400	-.1447	13.7400	-.1380
12	14.0000	13.5400	.4563	13.6500	.3503	13.4800	.5206	13.7200	.2786	13.7000	.3034
13	13.7000	13.8000	-.0980	13.7900	-.0868	13.9100	-.2107	13.7000	.0019	13.7900	-.0876
14	13.7000	13.7600	-.0603	13.7600	-.0555	13.8000	-.0979	13.6700	.0252	13.7600	-.0613
15	13.7000	13.7600	-.0629	13.7300	-.0326	13.7300	-.0311	13.6500	.0485	13.7400	-.0429
16	13.6000	13.7600	-.1640	13.7200	-.1192	13.6300	-.0300	13.6300	-.0282	13.7300	-.1301
17	13.7000	13.6000	.1019	13.6700	.0285	13.6200	.0771	13.6000	.0952	13.6900	.0090
18	13.7000	13.6400	.0619	13.6800	.0183	13.5400	.1580	13.5800	.1185	13.6900	.0063
19	13.7000	13.6600	.0376	13.6900	.0108	13.6200	.0753	13.5600	.1418	13.7000	.0044
20	13.6000	13.6800	-.0772	13.6900	-.0937	13.7600	-.1558	13.5300	.0651	13.7000	-.0969
21	13.7000	13.6500	.0523	13.6600	.0435	13.7300	-.0289	13.5100	.1884	13.6700	.0322
22	13.6000	13.6700	-.0679	13.6700	-.0729	13.5700	.0251	13.4900	.1117	13.6800	-.0775
23	13.7000	13.6200	.0838	13.6400	.0556	13.6700	.0343	13.4600	.2350	13.6500	.0458
24	13.6000	13.6500	-.0488	13.6700	-.0657	13.6400	-.0394	13.4400	.1583	13.6700	-.0680
25	13.7000	13.6300	.0698	13.6400	.0599	13.6500	.0513	13.4200	.2816	13.6500	.0524
26	14.0000	13.6600	.3428	13.6600	.3368	13.7100	.2934	13.4000	.6049	13.6600	.3367
27	13.6000	13.8900	-.2930	13.8000	-.1963	13.9300	-.3319	13.3700	.2283	13.7600	-.1643
28	13.6000	13.7500	-.1536	13.7200	-.1212	13.6700	-.0659	13.3500	.2516	13.7200	-.1150
29	13.6000	13.6900	-.0942	13.6700	-.0712	13.6600	-.0572	13.3300	.2749	13.6800	-.0805
30	13.5000	13.6300	-.1319	13.6400	-.1418	13.5000	.0022	13.3000	.1982	13.6600	-.1564
31	13.4000	13.4500	-.0538	13.5800	-.1848	13.4600	-.0572	13.2800	.1215	13.6100	-.2095
32	13.5000	13.3800	.1162	13.5100	-.0100	13.4500	.0503	13.2600	.2448	13.5500	-.0466
33	13.5000	13.4000	.0957	13.5000	-.0044	13.5400	-.0407	13.2300	.2681	13.5300	-.0326
34	13.1000	13.4200	-.3167	13.5000	-.4026	13.3700	-.2695	13.2100	-.1086	13.5200	-.4228
35	13.2000	13.1900	.0055	13.3400	-.1414	13.2300	-.0301	13.1900	.0147	13.4000	-.1960
36	12.5000	13.1500	-.6475	13.2800	-.7816	13.1000	-.6016	13.1600	-.6619	13.3400	-.8372
37	12.4000	12.6500	-.2510	12.9600	-.5648	12.6500	-.2505	13.1400	-.7386	13.0900	-.6860
38	12.6000	12.2900	.3136	12.7300	-.1309	12.3800	.2218	13.1200	-.5153	12.8800	-.2802
39	12.5000	12.2900	.2128	12.6700	-.1738	12.3400	.1587	13.0900	-.5920	12.8000	-.2962
40	12.5000	12.2000	.3022	12.6000	-.1035	12.2800	.2186	13.0700	-.5687	12.7100	-.2073
41	12.4000	12.3200	.0811	12.5600	-.1608	12.3600	.0393	13.0500	-.6454	12.6500	-.2451
42	12.3000	12.3500	-.0530	0.0000	0.0000	12.2100	.0943	13.0200	-.7221	12.5700	-.2716
43	12.4000	12.2600	.1418	12.4200	-.0164	12.1600	.2381	13.0000	-.5988	12.4900	-.0901
44	12.5000	12.2900	.2110	12.4100	.0918	12.3100	.1858	12.9800	-.4755	12.4600	.0369
45	12.6000	12.3700	.2284	12.4400	.1556	12.4700	.1312	12.9500	-.3521	12.4700	.1258
46	12.5000	12.5100	-.0121	12.5100	-.0067	12.4100	.0919	12.9300	-.4288	12.5100	-.0119
47	12.5000	12.5600	-.0591	12.5100	-.0054	12.5600	-.0589	12.9100	-.4055	12.5100	-.0083

Continuation Example of Output for Program COMBI

48	12.4000	12.5600	-.1616	12.5000	-.1032	12.4300	-.0269	12.8800	-.4822	12.5100	-.1058
49	12.6000	12.4700	.1270	12.4600	.1379	12.4700	.1278	12.8600	-.2589	12.4700	.1259
50	12.7000	12.5200	.1783	12.5200	.1843	12.6400	.0633	12.8400	-.1356	12.5100	.1881
51	12.7000	12.6400	.0580	12.5900	.1103	12.6500	.0486	12.8100	-.1123	12.5700	.1317
52	12.6000	12.7200	-.1157	12.6300	-.0350	12.6700	-.0682	12.7900	-.1890	12.6100	-.0078
53	12.7000	12.7200	-.0220	12.6200	.0779	12.6700	.0340	12.7700	-.0657	12.6100	.0945
54	12.7000	12.7400	-.0382	12.6500	.0474	12.6100	.0867	12.7400	-.0424	12.6300	.0662
55	12.5000	12.7200	-.2225	12.6700	-.1721	12.6800	-.1780	12.7200	-.2190	12.6500	-.1537
56	12.7000	12.5900	.1149	12.6000	.0960	12.6300	.0702	12.7000	.0043	12.6100	.0924
57	12.7000	12.6600	.0448	12.6400	.0596	12.7500	-.0505	12.6700	.0276	12.6400	.0647
58	12.6000	12.6700	-.0722	12.6600	-.0649	12.5900	.0074	12.6500	-.0491	12.6500	-.0547
59	12.7000	12.6200	.0813	12.6400	.0604	12.6900	.0120	12.6300	.0742	12.6400	.0617
60	12.7000	12.7000	-.0018	12.6600	.0371	12.6100	.0852	12.6000	.0975	12.6600	.0432
61	13.0000	12.7000	.2994	12.6800	.3218	12.7700	.2300	12.5800	.4208	12.6700	.3302
62	12.6000	12.8900	-.2935	12.8100	-.2052	13.0300	-.4339	12.5600	.0441	12.7700	-.1688
63	13.0000	12.7800	.2203	12.7300	.2737	12.7300	.2697	12.5300	.4675	12.7200	.2818
64	12.7000	12.9400	-.2417	12.8300	-.1322	12.8800	-.1778	12.5100	.1908	12.8000	-.1027
65	12.6000	12.8500	-.2474	12.7800	-.1821	12.8100	-.2132	12.4900	.1141	12.7700	-.1719
66	12.6000	12.6500	-.0493	12.7100	-.1084	12.5800	.0182	12.4600	.1374	12.7200	-.1203
67	12.5000	12.6300	-.1309	12.6600	-.1637	12.5400	-.0416	12.4400	.0607	12.6800	-.1842
68	12.5000	12.4500	.0462	12.6000	-.0976	12.5700	-.0683	12.4200	.0840	12.6300	-.1290
69	12.6000	12.4200	.1777	12.5600	.0427	12.5500	.0478	12.3900	.2073	12.5900	.0097
70	12.5000	12.4900	.0078	12.5700	-.0734	12.4300	.0720	12.3700	.1306	12.5900	-.0932
71	12.6000	12.4700	.1294	12.5400	.0554	12.5400	.0619	12.3500	.2539	12.5700	.0348
72	12.5000	12.5500	-.0470	12.5700	-.0659	12.4800	.0161	12.3200	.1773	12.5800	-.0757
73	12.4000	12.5300	-.1291	12.5400	-.1402	12.5800	-.1816	12.3000	.1006	12.5500	-.1530
74	12.5000	12.4300	.0715	12.4800	.0162	12.4700	.0318	12.2800	.2239	12.5100	-.0071
75	12.4000	12.4600	-.0566	12.4900	-.0890	12.4800	-.0798	12.2500	.1472	12.5000	-.1050
76	12.4000	12.3800	.0160	12.4500	-.0538	12.3200	.0770	12.2300	.1705	12.4700	-.0735
77	12.5000	12.3600	.1410	12.4300	.0684	12.3800	.1238	12.2100	.2938	12.4500	.0486
78	12.6000	12.3800	.2166	12.4600	.1418	12.3800	.2241	12.1800	.4171	12.4700	.1340
79	12.6000	12.4100	.1931	12.5100	.0851	12.4900	.1114	12.1600	.4404	12.5100	.0938
80	12.5000	12.4600	.0430	12.5500	-.0498	12.6600	-.1577	12.1400	.3637	12.5300	-.0343
81	12.5000	12.4800	.0163	12.5300	-.0308	12.6400	-.1395	12.1100	.3871	12.5200	-.0240
82	12.6000	12.4800	.1157	12.5200	.0819	12.4200	.1755	12.0900	.5104	12.5200	.0832
83	12.5000	12.4800	.0168	12.5500	-.0503	12.6300	-.1282	12.0700	.4337	12.5400	-.0418
84	12.7000	12.4600	.2415	12.5300	.1690	12.4700	.2344	12.0400	.6570	12.5300	.1708
85	12.5000	12.5100	-.0072	12.6000	-.0973	12.7100	-.2146	12.0200	.4803	12.5800	-.0805
86	12.7000	12.5100	.1911	12.5600	.1398	12.6300	.0694	12.0000	.7036	12.5600	.1437
87	12.7000	12.5300	.1674	12.6100	.0655	12.6900	.0079	11.9700	.7269	12.6000	.1006
88	12.7000	12.5800	.1174	12.6500	.0504	12.6500	.0544	11.9500	.7502	12.6300	.0704
89	12.6000	12.5800	.0177	12.6700	-.0703	12.7300	-.1337	11.9300	.6735	12.6500	-.0507
90	12.6000	12.6100	-.0091	12.6400	-.0428	12.6000	.0012	11.9000	.6968	12.6400	-.0355
91	12.6000	12.5800	.0157	12.6300	-.0252	12.5500	.0454	11.8800	.7202	12.6200	-.0249
92	12.6000	12.5600	.0409	12.6100	-.0148	12.6400	-.0408	11.8600	.7435	12.6200	-.0174
93	12.6000	12.5300	.0663	12.6100	-.0087	12.6900	-.0866	11.8300	.7668	12.6100	-.0122
94	12.7000	12.5300	.1662	12.6100	.0949	12.5300	.1738	11.8100	.8901	12.6100	.0915
95	12.7000	12.5600	.1421	12.6400	.0574	12.7000	-.0032	11.7900	.9134	12.6400	.0640
96	12.7000	12.5800	.1172	12.6700	.0338	12.6600	.0431	11.7600	.9367	12.6600	.0448
97	12.7000	12.6100	.0919	12.6800	.0199	12.7500	-.0454	11.7400	.9600	12.6700	.0314
98	12.7000	12.6300	.0664	12.6900	.0117	12.8000	-.0968	11.7200	.9833	12.6800	.0220
99	13.0000	12.6300	.3664	12.6900	.3069	12.7400	.2557	11.6900	1.3070	12.6800	.3154
100	12.5000	12.7100	-.2059	12.8100	-.3140	12.8700	-.3694	11.6700	.8300	12.7800	-.2792

Continuation Example of Output for Program COMBI

```
SUMMARY OF STATISTICAL VALUES:(FIRST   75 POINTS OF DATA CONSIDERED)
METHOD:                          ADAPTIVE       BOX JENKIN
(1) SUM OF ERRORS:                -.0946          -3.5676
(2) MEANS:                        -.0014           -.0495
(3) SUM(ERROR-MEAN):               .0014           -.0495
(4) VARIANCE:                      .0330            .0314
(5) STANDARD DEVIATION:            .1815            .1771
(6) MEAN SQUARE ERROR:             .0330            .0340

NUMBER OF ERRORS USED IN COMBINATION:   70

CORRELATION COEFFICIENTS:
                       ADAPTIVE        BOX JENK
ADAPTIVE                1.0000
BOX JENK                 .7974          1.0000

COVARIANCE MATRIX:
                       ADAPTIVE        BOX JENK
ADAPTIVE                 .0330           .0256
BOX JENK                 .0256           .0314

INVERSE OF COVARIANCE MATRIX:
                       ADAPTIVE        BOX JENK
ADAPTIVE               83.3108         -68.0742
BOX JENK              -68.0742          87.4899

WEIGHT VECTOR FOR COMBINED FORECASTS
     .4397E+00
     .5603E+00

COMBINATION OF   2 METHODS ADAPTIVE    BOX JENKIN           ERROR VARIANCE=        .0289
                                                            % IMPROVEMENT OVER MIN. VARIANCE:  8.04
```

Continuation Example of Output for Program COMBI

```
SUMMARY OF STATISTICAL VALUES:(FIRST   75 POINTS OF DATA CONSIDERED)
METHOD:                          ADAPTIVE        WINTERS

(1) SUM OF ERRORS:                -.0946          -.2334

(2) MEANS:                        -.0014          -.0032

(3) SUM(ERROR-MEAN):               .0014          -.0032

(4) VARIANCE:                      .0334           .0289

(5) STANDARD DEVIATION:            .1828           .1699

(6) MEAN SQUARE ERROR:             .0330           .0289

NUMBER OF ERRORS USED IN COMBINATION:   70

CORRELATION COEFFICIENTS:
                        ADAPTIVE          WINTERS
ADAPTIVE                  1.0000
WINTERS                    .8981           1.0000

COVARIANCE MATRIX:
                        ADAPTIVE          WINTERS
ADAPTIVE                   .0334            .0279
WINTERS                    .0279            .0289

INVERSE OF COVARIANCE MATRIX:
                        ADAPTIVE          WINTERS
ADAPTIVE                154.6223        -149.4265
WINTERS                -149.4265         179.0385

WEIGHT VECTOR FOR COMBINED FORECASTS
      .1493E+00
      .8507E+00

COMBINATION OF    2 METHODS ADAPTIVE    WINTERS          ERROR VARIANCE=       .0287
                                                         % IMPROVEMENT OVER MIN. VARIANCE:    .50
```

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	REGRESSION
(1) SUM OF ERRORS:	-.0946	-2.0776
(2) MEANS:	-.0014	-.0289
(3) SUM(ERROR-MEAN):	.0014	-.0289
(4) VARIANCE:	.0334	.0908
(5) STANDARD DEVIATION:	.1829	.3013
(6) MEAN SQUARE ERROR:	.0330	.0917

NUMBER OF ERRORS USED IN COMBINATION: 70

CORRELATION COEFFICIENTS:

	ADAPTIVE	REGRESSI
ADAPTIVE	1.0000	
REGRESSI	.1507	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	REGRESSI
ADAPTIVE	.0334	.0083
REGRESSI	.0083	.0908

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	REGRESSI
ADAPTIVE	30.5984	-2.7981
REGRESSI	-2.7981	11.2721

WEIGHT VECTOR FOR COMBINED FORECASTS

.7664E+00
.2336E+00

COMBINATION OF 2 METHODS ADAPTIVE REGRESSION

ERROR VARIANCE= .0276
% IMPROVEMENT OVER MIN. VARIANCE: 17.56

Continuation Example of Output for Program COMBI

```
SUMMARY OF STATISTICAL VALUES:(FIRST   75 POINTS OF DATA CONSIDERED)
METHOD:                              ADAPTIVE        SMOOTHING

 (1) SUM OF ERRORS:                    .0651          -4.7496

 (2) MEANS:                            .0009           -.0660

 (3) SUM(ERROR-MEAN):                  .0009           -.1979

 (4) VARIANCE:                         .0327            .0364

 (5) STANDARD DEVIATION:               .1808            .1907

 (6) MEAN SQUARE ERROR:                .0322            .0413

NUMBER OF ERRORS USED IN COMBINATION:   68

CORRELATION COEFFICIENTS:
                   ADAPTIVE      SMOOTHIN
ADAPTIVE             1.0000
SMOOTHIN              .6517        1.0000

COVARIANCE MATRIX:
                   ADAPTIVE      SMOOTHIN
ADAPTIVE              .0327         .0225
SMOOTHIN              .0225         .0364

INVERSE OF COVARIANCE MATRIX:
                   ADAPTIVE      SMOOTHIN
ADAPTIVE            53.1858      -32.8535
SMOOTHIN           -32.8535       47.7843

WEIGHT VECTOR FOR COMBINED FORECASTS
      .5766E+00
      .4234E+00

COMBINATION OF    2 METHODS ADAPTIVE    SMOOTHING        ERROR VARIANCE=         .0284
                                                         % IMPROVEMENT OVER MIN. VARIANCE: 13.23
```

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	BOX JENKIN	WINTERS
(1) SUM OF ERRORS:	-3.5700	-.1829
(2) MEANS:	-.0510	-.0025
(3) SUM(ERROR-MEAN):	.1530	.0025
(4) VARIANCE:	.0310	.0282
(5) STANDARD DEVIATION:	.1761	.1678
(6) MEAN SQUARE ERROR:	.0330	.0282

NUMBER OF ERRORS USED IN COMBINATION: 72

CORRELATION COEFFICIENTS:

	BOX JENK	WINTERS
BOX JENK	1.0000	
WINTERS	.7779	1.0000

COVARIANCE MATRIX:

	BOX JENK	WINTERS
BOX JENK	.0310	.0230
WINTERS	.0230	.0282

INVERSE OF COVARIANCE MATRIX:

	BOX JENK	WINTERS
BOX JENK	81.6389	-66.6659
WINTERS	-66.6659	89.9560

WEIGHT VECTOR FOR COMBINED FORECASTS
.3913E+00
.6087E+00

COMBINATION OF 2 METHODS BOX JENKIN WINTERS

ERROR VARIANCE= .0261
% IMPROVEMENT OVER MIN. VARIANCE: 7.18

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	BOX JENKIN	REGRESSION
(1) SUM OF ERRORS:	-3.5700	-1.9167
(2) MEANS:	-.0510	-.0266
(3) SUM(ERROR-MEAN):	.1530	.0266
(4) VARIANCE:	.0310	.0886
(5) STANDARD DEVIATION:	.1761	.2976
(6) MEAN SQUARE ERROR:	.0330	.0893

NUMBER OF ERRORS USED IN COMBINATION: 72

CORRELATION COEFFICIENTS:

	BOX JENK	REGRESSI
BOX JENK	1.0000	
REGRESSI	.4423	1.0000

COVARIANCE MATRIX:

	BOX JENK	REGRESSI
BOX JENK	.0310	.0232
REGRESSI	.0232	.0886

INVERSE OF COVARIANCE MATRIX:

	BOX JENK	REGRESSI
BOX JENK	40.1028	-10.4950
REGRESSI	-10.4950	14.0395

WEIGHT VECTOR FOR COMBINED FORECASTS
.8931E+00
.1069E+00

COMBINATION OF 2 METHODS BOX JENKIN REGRESSION

ERROR VARIANCE= .0302
% IMPROVEMENT OVER MIN. VARIANCE: 2.70

Continuation Example of Output for Program COMBI

```
SUMMARY OF STATISTICAL VALUES:(FIRST   75 POINTS OF DATA CONSIDERED)
METHOD:                             WINTERS      REGRESSION
(1) SUM OF ERRORS:                   -.2650       -1.8712
(2) MEANS:                           -.0038        -.0260
(3) SUM(ERROR-MEAN):                  .0151         .0520
(4) VARIANCE:                         .0283         .0874
(5) STANDARD DEVIATION:               .1682         .2956
(6) MEAN SQUARE ERROR:                .0279         .0880

NUMBER OF ERRORS USED IN COMBINATION:   73

CORRELATION COEFFICIENTS:
                    WINTERS         REGRESSI
WINTERS              1.0000
REGRESSI              .1083           1.0000

COVARIANCE MATRIX:
                    WINTERS         REGRESSI
WINTERS               .0283            .0054
REGRESSI              .0054            .0874

INVERSE OF COVARIANCE MATRIX:
                    WINTERS         REGRESSI
WINTERS             35.7747          -2.2041
REGRESSI            -2.2041          11.5785

WEIGHT VECTOR FOR COMBINED FORECASTS
    .7817E+00
    .2183E+00

COMBINATION OF    2 METHODS WINTERS     REGRESSION           ERROR VARIANCE=        .0233
                                                             % IMPROVEMENT OVER MIN. VARIANCE: 17.67
```

Continuation Example of Output for Program COMBI

```
SUMMARY OF STATISTICAL VALUES:(FIRST   75 POINTS OF DATA CONSIDERED)
METHOD:                          WINTERS          SMOOTHING
(1) SUM OF ERRORS:               -.0546           -4.7496
(2) MEANS:                       -.0008           -.0660
(3) SUM(ERROR-MEAN):             -.0008           -.1979
(4) VARIANCE:                     .0288            .0364
(5) STANDARD DEVIATION:           .1698            .1907
(6) MEAN SQUARE ERROR:            .0284            .0413

NUMBER OF ERRORS USED IN COMBINATION:   68

CORRELATION COEFFICIENTS:
                     WINTERS          SMOOTHIN
WINTERS               1.0000
SMOOTHIN               .6476           1.0000

COVARIANCE MATRIX:
                     WINTERS          SMOOTHIN
WINTERS                .0288            .0210
SMOOTHIN               .0210            .0364

INVERSE OF COVARIANCE MATRIX:
                     WINTERS          SMOOTHIN
WINTERS              59.7241          -34.4327
SMOOTHIN            -34.4327           47.3418

WEIGHT VECTOR FOR COMBINED FORECASTS
     .6621E+00
     .3379E+00

COMBINATION OF    2 METHODS WINTERS     SMOOTHING                          ERROR VARIANCE=        .0262
                                                                           % IMPROVEMENT OVER MIN. VARIANCE:  9.21

BEST COMBINATION OCCURS FOR    2 METHODS WINTERS     REGRESSION
FOR WHICH VARIANCE OF THE COMBINED FORECAST ERROR IS         .0233
```

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES: (FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	BOX JENKIN	WINTERS
(1) SUM OF ERRORS:	-.0946	-3.5676	-.2334
(2) MEANS:	-.0014	-.0495	-.0032
(3) SUM(ERROR-MEAN):	.0014	-.0495	-.0064
(4) VARIANCE:	.0334	.0319	.0289
(5) STANDARD DEVIATION:	.1827	.1786	.1699
(6) MEAN SQUARE ERROR:	.0330	.0340	.0289

NUMBER OF ERRORS USED IN COMBINATION: 70

CORRELATION COEFFICIENTS:

	ADAPTIVE	BOX JENK	WINTERS
ADAPTIVE	1.0000		
BOX JENK	.7858	1.0000	
WINTERS	.9087	.7781	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS
ADAPTIVE	.0334	.0256	.0282
BOX JENK	.0256	.0319	.0236
WINTERS	.0282	.0236	.0289

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS
ADAPTIVE	188.9398	-38.5567	-153.0414
BOX JENK	-38.5567	87.2970	-33.7395
WINTERS	-153.0414	-33.7395	211.7408

WEIGHT VECTOR FOR COMBINED FORECASTS

-.7127E-01
.4021E+00
.6691E+00

COMBINATION OF 3 METHODS ADAPTIVE BOX JENKIN WINTERS

ERROR VARIANCE= .0268
% IMPROVEMENT OVER MIN. VARIANCE: 7.16

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	BOX JENKIN	REGRESSION
(1) SUM OF ERRORS:	-.0946	-3.5676	-2.0776
(2) MEANS:	-.0014	-.0495	-.0285
(3) SUM(ERROR-MEAN):	.0014	-.0495	-.0569
(4) VARIANCE:	.0334	.0318	.0908
(5) STANDARD DEVIATION:	.1829	.1784	.3013
(6) MEAN SQUARE ERROR:	.0330	.0340	.0917

NUMBER OF ERRORS USED IN COMBINATION: 70

CORRELATION COEFFICIENTS:

	ADAPTIVE	BOX JENK	REGRESSI
ADAPTIVE	1.0000		
BOX JENK	.7858	1.0000	
REGRESSI	.1581	.4407	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	REGRESSI
ADAPTIVE	.0334	.0256	.0087
BOX JENK	.0256	.0318	.0237
REGRESSI	.0087	.0237	.0908

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	REGRESSI
ADAPTIVE	88.3308	-80.4457	12.5227
BOX JENK	-80.4457	112.2362	-21.5772
REGRESSI	12.5227	-21.5772	15.4468

WEIGHT VECTOR FOR COMBINED FORECASTS
.5514E+00
.2759E+00
.1727E+00

COMBINATION OF 3 METHODS ADAPTIVE BOX JENKIN REGRESSION

ERROR VARIANCE= .0270
% IMPROVEMENT OVER MIN. VARIANCE: 15.16

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	WINTERS	REGRESSION
(1) SUM OF ERRORS:	-.0946	-.2334	-2.0776
(2) MEANS:	-.0014	-.0032	-.0285
(3) SUM(ERROR-MEAN):	.0014	-.0032	-.0569
(4) VARIANCE:	.0334	.0293	.0908
(5) STANDARD DEVIATION:	.1828	.1712	.3013
(6) MEAN SQUARE ERROR:	.0330	.0289	.0917

NUMBER OF ERRORS USED IN COMBINATION: 70

CORRELATION COEFFICIENTS:

	ADAPTIVE	WINTERS	REGRESSI
ADAPTIVE	1.0000		
WINTERS	.8912	1.0000	
REGRESSI	.1566	.1084	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	WINTERS	REGRESSI
ADAPTIVE	.0334	.0279	.0086
WINTERS	.0279	.0293	.0056
REGRESSI	.0086	.0056	.0908

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	WINTERS	REGRESSI
ADAPTIVE	147.9848	-139.7668	-5.4523
WINTERS	-139.7668	166.5097	3.0230
REGRESSI	-5.4523	3.0230	11.3482

WEIGHT VECTOR FOR COMBINED FORECASTS

.6672E-01
.7181E+00
.2152E+00

COMBINATION OF 3 METHODS ADAPTIVE WINTERS REGRESSION ERROR VARIANCE= .0241
% IMPROVEMENT OVER MIN. VARIANCE: 17.74

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	BOX JENKIN	SMOOTHING
(1) SUM OF ERRORS:	.0651	-3.3272	-4.7496
(2) MEANS:	.0009	-.0462	-.0651
(3) SUM(ERROR-MEAN):	.0009	-.1386	-.2603
(4) VARIANCE:	.0327	.0312	.0364
(5) STANDARD DEVIATION:	.1808	.1766	.1907
(6) MEAN SQUARE ERROR:	.0322	.0331	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	ADAPTIVE	BOX JENK	SMOOTHIN
ADAPTIVE	1.0000		
BOX JENK	.7746	1.0000	
SMOOTHIN	.6555	.9503	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	SMOOTHIN
ADAPTIVE	.0327	.0247	.0226
BOX JENK	.0247	.0312	.0320
SMOOTHIN	.0226	.0320	.0364

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	SMOOTHIN
ADAPTIVE	91.9348	-147.3152	72.5409
BOX JENK	-147.3152	566.7839	-407.3271
SMOOTHIN	72.5409	-407.3271	340.9286

WEIGHT VECTOR FOR COMBINED FORECASTS

.4841E+00
.3426E+00
.1733E+00

COMBINATION OF 3 METHODS ADAPTIVE BOX JENKIN SMOOTHING

ERROR VARIANCE= .0282
% IMPROVEMENT OVER MIN. VARIANCE: 9.58

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	WINTERS	SMOOTHING
(1) SUM OF ERRORS:	.0651	-.0546	-4.7496
(2) MEANS:	.0009	-.0008	-.0651
(3) SUM(ERROR-MEAN):	.0009	-.0023	-.2603
(4) VARIANCE:	.0327	.0289	.0364
(5) STANDARD DEVIATION:	.1808	.1699	.1907
(6) MEAN SQUARE ERROR:	.0322	.0284	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	ADAPTIVE	WINTERS	SMOOTHIN
ADAPTIVE	1.0000		
WINTERS	.8864	1.0000	
SMOOTHIN	.6554	.6474	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	WINTERS	SMOOTHIN
ADAPTIVE	.0327	.0272	.0226
WINTERS	.0272	.0289	.0210
SMOOTHIN	.0226	.0210	.0364

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	WINTERS	SMOOTHIN
ADAPTIVE	150.8706	-127.7389	-20.0835
WINTERS	-127.7389	167.8227	-17.3965
SMOOTHIN	-20.0835	-17.3965	49.9970

WEIGHT VECTOR FOR COMBINED FORECASTS

.7968E-01
.5931E+00
.3272E+00

COMBINATION OF 3 METHODS ADAPTIVE WINTERS SMOOTHING

ERROR VARIANCE= .0261
% IMPROVEMENT OVER MIN. VARIANCE: 9.39

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	REGRESSION	SMOOTHING
(1) SUM OF ERRORS:	.0651	-2.1317	-4.7496
(2) MEANS:	.0009	-.0296	-.0651
(3) SUM(ERROR-MEAN):	.0009	-.0888	-.2603
(4) VARIANCE:	.0327	.0928	.0364
(5) STANDARD DEVIATION:	.1808	.3046	.1907
(6) MEAN SQUARE ERROR:	.0322	.0933	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	ADAPTIVE	REGRESSI	SMOOTHIN
ADAPTIVE	1.0000		
REGRESSI	.1344	1.0000	
SMOOTHIN	.6616	.5461	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	REGRESSI	SMOOTHIN
ADAPTIVE	.0327	.0074	.0228
REGRESSI	.0074	.0928	.0317
SMOOTHIN	.0228	.0317	.0364

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	REGRESSI	SMOOTHIN
ADAPTIVE	62.6042	12.0143	-49.7287
REGRESSI	12.0143	17.6674	-22.9393
SMOOTHIN	-49.7287	-22.9393	78.6733

WEIGHT VECTOR FOR COMBINED FORECASTS

.6613E+00
.1791E+00
.1596E+00

COMBINATION OF 3 METHODS ADAPTIVE REGRESSION SMOOTHING ERROR VARIANCE= .0266
% IMPROVEMENT OVER MIN. VARIANCE: 18.68

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	BOX JENKIN	WINTERS	REGRESSION
(1) SUM OF ERRORS:	-3.5700	-.1829	-1.9167
(2) MEANS:	-.0510	-.0025	-.0263
(3) SUM(ERROR-MEAN):	.1530	.0025	-.0000
(4) VARIANCE:	.0310	.0295	.0886
(5) STANDARD DEVIATION:	.1761	.1716	.2976
(6) MEAN SQUARE ERROR:	.0330	.0282	.0893

NUMBER OF ERRORS USED IN COMBINATION: 72

CORRELATION COEFFICIENTS:

	BOX JENK	WINTERS	REGRESSI
BOX JENK	1.0000		
WINTERS	.7605	1.0000	
REGRESSI	.4483	.1085	1.0000

COVARIANCE MATRIX:

	BOX JENK	WINTERS	REGRESSI
BOX JENK	.0310	.0230	.0235
WINTERS	.0230	.0295	.0055
REGRESSI	.0235	.0055	.0886

INVERSE OF COVARIANCE MATRIX:

	BOX JENK	WINTERS	REGRESSI
BOX JENK	112.6005	-83.2333	-24.6696
WINTERS	-83.2333	95.8712	16.0870
REGRESSI	-24.6696	16.0870	16.8322

WEIGHT VECTOR FOR COMBINED FORECASTS

.1127E+00
.6893E+00
.1980E+00

COMBINATION OF 3 METHODS BOX JENKIN WINTERS REGRESSION

ERROR VARIANCE= .0240
% IMPROVEMENT OVER MIN. VARIANCE: 18.55

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	BOX JENKIN	WINTERS	SMOOTHING
(1) SUM OF ERRORS:	-3.3272	-.0546	-4.7496
(2) MEANS:	-.0475	-.0008	-.0651
(3) SUM(ERROR-MEAN):	-.0475	-.0023	-.2603
(4) VARIANCE:	.0312	.0289	.0364
(5) STANDARD DEVIATION:	.1766	.1699	.1907
(6) MEAN SQUARE ERROR:	.0331	.0284	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	BOX JENK	WINTERS	SMOOTHIN
BOX JENK	1.0000		
WINTERS	.7628	1.0000	
SMOOTHIN	.9609	.6474	1.0000

COVARIANCE MATRIX:

	BOX JENK	WINTERS	SMOOTHIN
BOX JENK	.0312	.0229	.0324
WINTERS	.0229	.0289	.0210
SMOOTHIN	.0324	.0210	.0364

INVERSE OF COVARIANCE MATRIX:

	BOX JENK	WINTERS	SMOOTHIN
BOX JENK	753.2332	-189.6920	-560.8543
WINTERS	-189.6920	107.4331	106.8472
SMOOTHIN	-560.8543	106.8472	464.9309

WEIGHT VECTOR FOR COMBINED FORECASTS

.7034E-01
.6437E+00
.2860E+00

COMBINATION OF 3 METHODS BOX JENKIN WINTERS SMOOTHING

ERROR VARIANCE= .0262
% IMPROVEMENT OVER MIN. VARIANCE: 9.27

Continuation Example of Output for Program COMBI

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SUMMARY OF STATISTICAL VALUES:(FIRST   75 POINTS OF DATA CONSIDERED)
METHOD:                        WINTERS      REGRESSION     SMOOTHING

(1) SUM OF ERRORS:             -.0546       -2.1317        -4.7496

(2) MEANS:                     -.0008       -.0296         -.0651

(3) SUM(ERROR-MEAN):           -.0008       -.0888         -.2603

(4) VARIANCE:                  .0289        .0928          .0364

(5) STANDARD DEVIATION:        .1699        .3046          .1907

(6) MEAN SQUARE ERROR:         .0284        .0933          .0413

NUMBER OF ERRORS USED IN COMBINATION:   68

CORRELATION COEFFICIENTS:
                  WINTERS        REGRESSI        SMOOTHIN
WINTERS           1.0000
REGRESSI          .0934          1.0000
SMOOTHIN          .6620          .5461           1.0000

COVARIANCE MATRIX:
                  WINTERS        REGRESSI        SMOOTHIN
WINTERS           .0289          .0048           .0215
REGRESSI          .0048          .0928           .0317
SMOOTHIN          .0215          .0317           .0364

INVERSE OF COVARIANCE MATRIX:
                  WINTERS        REGRESSI        SMOOTHIN
WINTERS           75.3826        16.0687         -58.4738
REGRESSI          16.0687        18.7870         -25.8603
SMOOTHIN          -58.4738       -25.8603        84.5299

WEIGHT VECTOR FOR COMBINED FORECASTS
       .7820E+00
       .2133E+00
       .4641E-02

COMBINATION OF   3 METHODS WINTERS   REGRESSION  SMOOTHING        ERROR VARIANCE=      .0237
                                                                  % IMPROVEMENT OVER MIN, VARIANCE: 17.88
```

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES: (FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	BOX JENKIN	WINTERS	REGRESSION
(1) SUM OF ERRORS:	-.0946	-3.5676	-.2334	-2.0776
(2) MEANS:	-.0014	-.0495	-.0032	-.0281
(3) SUM(ERROR-MEAN):	.0014	-.0495	-.0064	-.0842
(4) VARIANCE:	.0334	.0327	.0294	.0908
(5) STANDARD DEVIATION:	.1827	.1809	.1715	.3013
(6) MEAN SQUARE ERROR:	.0330	.0340	.0289	.0917

NUMBER OF ERRORS USED IN COMBINATION: 70

CORRELATION COEFFICIENTS:

	ADAPTIVE	BOX JENK	WINTERS	REGRESSI
ADAPTIVE	1.0000			
BOX JENK	.7759	1.0000		
WINTERS	.9007	.7614	1.0000	
REGRESSI	.1521	.4379	.1083	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS	REGRESSI
ADAPTIVE	.0334	.0256	.0282	.0084
BOX JENK	.0256	.0327	.0236	.0239
WINTERS	.0282	.0236	.0294	.0056
REGRESSI	.0084	.0239	.0056	.0908

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS	REGRESSI
ADAPTIVE	177.5870	-43.2170	-136.3489	3.3891
BOX JENK	-43.2170	115.0041	-46.4603	-23.3868
WINTERS	-136.3489	-46.4603	199.8067	12.4740
REGRESSI	3.3891	-23.3868	12.4740	16.0835

WEIGHT VECTOR FOR COMBINED FORECASTS

.3408E-01
.4688E-01
.7122E+00
.2069E+00

COMBINATION OF 4 METHODS ADAPTIVE BOX JENKIN WINTERS REGRESSION ERROR VARIANCE= .0242
% IMPROVEMENT OVER MIN. VARIANCE: 17.81

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	BOX JENKIN	WINTERS	SMOOTHING
(1) SUM OF ERRORS:	.0651	-3.3272	-.0546	-4.7496
(2) MEANS:	.0009	-.0462	-.0007	-.0642
(3) SUM(ERROR-MEAN):	.0009	-.1386	-.0030	-.3209
(4) VARIANCE:	.0327	.0312	.0289	.0364
(5) STANDARD DEVIATION:	.1808	.1767	.1699	.1907
(6) MEAN SQUARE ERROR:	.0322	.0331	.0284	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	ADAPTIVE	BOX JENK	WINTERS	SMOOTHIN
ADAPTIVE	1.0000			
BOX JENK	.7745	1.0000		
WINTERS	.9001	.7625	1.0000	
SMOOTHIN	.6553	.9607	.6474	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS	SMOOTHIN
ADAPTIVE	.0327	.0247	.0276	.0226
BOX JENK	.0247	.0312	.0229	.0324
WINTERS	.0276	.0229	.0289	.0210
SMOOTHIN	.0226	.0324	.0210	.0364

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS	SMOOTHIN
ADAPTIVE	191.0815	-135.0847	-132.6045	77.9777
BOX JENK	-135.0847	839.1201	-93.3963	-609.0121
WINTERS	-132.6045	-93.3963	198.7844	50.8788
SMOOTHIN	77.9777	-609.0121	50.8788	491.7052

WEIGHT VECTOR FOR COMBINED FORECASTS

.3585E-01
.4258E-01
.6193E+00
.3023E+00

COMBINATION OF 4 METHODS ADAPTIVE BOX JENKIN WINTERS SMOOTHING ERROR VARIANCE= .0262
% IMPROVEMENT OVER MIN. VARIANCE: 9.29

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	BOX JENKIN	REGRESSION	SMOOTHING
(1) SUM OF ERRORS:	.0651	-3.3272	-2.1317	-4.7496
(2) MEANS:	.0009	-.0462	-.0292	-.0642
(3) SUM(ERROR-MEAN):	.0009	-.1386	-.1168	-.3209
(4) VARIANCE:	.0327	.0312	.0928	.0364
(5) STANDARD DEVIATION:	.1808	.1766	.3046	.1907
(6) MEAN SQUARE ERROR:	.0322	.0331	.0933	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	ADAPTIVE	BOX JENK	REGRESSI	SMOOTHIN
ADAPTIVE	1.0000			
BOX JENK	.7749	1.0000		
REGRESSI	.1418	.4329	1.0000	
SMOOTHIN	.6616	.9648	.5461	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	REGRESSI	SMOOTHIN
ADAPTIVE	.0327	.0247	.0078	.0228
BOX JENK	.0247	.0312	.0233	.0325
REGRESSI	.0078	.0233	.0928	.0317
SMOOTHIN	.0228	.0325	.0317	.0364

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	REGRESSI	SMOOTHIN
ADAPTIVE	105.3192	-204.1900	3.6926	113.1757
BOX JENK	-204.1900	962.4645	36.8260	-763.9835
REGRESSI	3.6926	36.8260	18.9059	-51.7031
SMOOTHIN	113.1757	-763.9835	-51.7031	684.1833

WEIGHT VECTOR FOR COMBINED FORECASTS

.4674E+00
.8081E+00
.2005E+00
-.4759E+00

COMBINATION OF 4 METHODS ADAPTIVE BOX JENKIN REGRESSION SMOOTHING ERROR VARIANCE= .0260
% IMPROVEMENT OVER MIN. VARIANCE: 16.75

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	WINTERS	REGRESSION	SMOOTHING
(1) SUM OF ERRORS:	.0651	-.0546	-2.1317	-4.7496
(2) MEANS:	.0009	-.0008	-.0292	-.0642
(3) SUM(ERROR-MEAN):	.0009	-.0023	-.1168	-.3209
(4) VARIANCE:	.0327	.0289	.0937	.0364
(5) STANDARD DEVIATION:	.1808	.1699	.3061	.1907
(6) MEAN SQUARE ERROR:	.0322	.0284	.0933	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	ADAPTIVE	WINTERS	REGRESSI	SMOOTHIN
ADAPTIVE	1.0000			
WINTERS	.8861	1.0000		
REGRESSI	.1358	.0929	1.0000	
SMOOTHIN	.6617	.6620	.5433	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	WINTERS	REGRESSI	SMOOTHIN
ADAPTIVE	.0327	.0272	.0075	.0228
WINTERS	.0272	.0289	.0048	.0215
REGRESSI	.0075	.0048	.0937	.0317
SMOOTHIN	.0228	.0215	.0317	.0364

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	WINTERS	REGRESSI	SMOOTHIN
ADAPTIVE	149.6563	-125.5541	1.6675	-21.2394
WINTERS	-125.5541	180.4915	14.3926	-40.2850
REGRESSI	1.6675	14.3926	18.4764	-25.6474
SMOOTHIN	-21.2394	-40.2850	-25.6474	86.9373

WEIGHT VECTOR FOR COMBINED FORECASTS

.1073E+00
.6878E+00
.2105E+00
-.5554E-02

COMBINATION OF 4 METHODS ADAPTIVE WINTERS REGRESSION SMOOTHING ERROR VARIANCE= .0237
% IMPROVEMENT OVER MIN. VARIANCE: 18.00

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES: (FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	BOX JENKIN	WINTERS	REGRESSION	SMOOTHING
(1) SUM OF ERRORS:	-3.3272	-.0546	-2.1317	-4.7496
(2) MEANS:	-.0475	-.0008	-.0292	-.0642
(3) SUM(ERROR-MEAN):	-.0475	-.0023	-.1168	-.3209
(4) VARIANCE:	.0312	.0288	.0937	.0364
(5) STANDARD DEVIATION:	.1767	.1698	.3061	.1907
(6) MEAN SQUARE ERROR:	.0331	.0284	.0933	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	BOX JENK	WINTERS	REGRESSI	SMOOTHIN
BOX JENK	1.0000			
WINTERS	.7626	1.0000		
REGRESSI	.4326	.0930	1.0000	
SMOOTHIN	.9602	.6573	.5433	1.0000

COVARIANCE MATRIX:

	BOX JENK	WINTERS	REGRESSI	SMOOTHIN
BOX JENK	.0312	.0229	.0234	.0324
WINTERS	.0229	.0288	.0048	.0213
REGRESSI	.0234	.0048	.0937	.0317
SMOOTHIN	.0324	.0213	.0317	.0364

INVERSE OF COVARIANCE MATRIX:

	BOX JENK	WINTERS	REGRESSI	SMOOTHIN
BOX JENK	693.1240	-150.6995	19.2094	-545.0813
WINTERS	-150.6995	106.7016	11.1941	61.8307
REGRESSI	19.2094	11.1941	18.8644	-40.0896
SMOOTHIN	-545.0813	61.8307	-40.0896	511.1128

WEIGHT VECTOR FOR COMBINED FORECASTS

.3892E+00
.6825E+00
.2158E+00
-.2875E+00

COMBINATION OF 4 METHODS BOX JENKIN WINTERS REGRESSION SMOOTHING

ERROR VARIANCE= .0235
% IMPROVEMENT OVER MIN. VARIANCE: 18.48

Continuation Example of Output for Program COMBI

SUMMARY OF STATISTICAL VALUES:(FIRST 75 POINTS OF DATA CONSIDERED)

METHOD:	ADAPTIVE	BOX JENKIN	WINTERS	REGRESSION	SMOOTHING
(1) SUM OF ERRORS:	.0651	-3.3272	-.0546	-2.1317	-4.7496
(2) MEANS:	.0009	-.0462	-.0007	-.0288	-.0698
(3) SUM(ERROR-MEAN):	.0009	-.1386	-.0030	-.1440	.0698
(4) VARIANCE:	.0326	.0312	.0298	.0929	.0364
(5) STANDARD DEVIATION:	.1807	.1765	.1727	.3047	.1907
(6) MEAN SQUARE ERROR:	.0322	.0331	.0284	.0933	.0413

NUMBER OF ERRORS USED IN COMBINATION: 68

CORRELATION COEFFICIENTS:

	ADAPTIVE	BOX JENK	WINTERS	REGRESSI	SMOOTHIN
ADAPTIVE	1.0000				
BOX JENK	.7756	1.0000			
WINTERS	.8836	.7508	1.0000		
REGRESSI	.1431	.4388	.0919	1.0000	
SMOOTHIN	.6619	.9455	.6391	.5458	1.0000

COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS	REGRESSI	SMOOTHIN
ADAPTIVE	.0326	.0247	.0276	.0079	.0228
BOX JENK	.0247	.0312	.0229	.0236	.0318
WINTERS	.0276	.0229	.0298	.0048	.0210
REGRESSI	.0079	.0236	.0048	.0929	.0317
SMOOTHIN	.0228	.0318	.0210	.0317	.0364

INVERSE OF COVARIANCE MATRIX:

	ADAPTIVE	BOX JENK	WINTERS	REGRESSI	SMOOTHIN
ADAPTIVE	165.7626	-86.1342	-111.8403	1.8630	34.5090
BOX JENK	-86.1342	507.3746	-54.2655	5.2032	-363.1244
WINTERS	-111.8403	-54.2655	170.1834	11.2231	9.3715
REGRESSI	1.8630	5.2032	11.2231	18.3369	-28.2052
SMOOTHIN	34.5090	-363.1244	9.3715	-28.2052	342.7898

WEIGHT VECTOR FOR COMBINED FORECASTS

.9989E-01
.2174E+00
.5924E+00
.2022E+00
-.1119E+00

COMBINATION OF 5 METHODS ADAPTIVE BOX JENKIN WINTERS REGRESSION SMOOTHING ERROR VARIANCE= .0240
% IMPROVEMENT OVER MIN. VARIANCE: 19.46

BEST COMBINATION OCCURS FOR 2 METHODS WINTERS REGRESSION
FOR WHICH VARIANCE OF THE COMBINED FORECAST ERROR IS .0233

Continuation Example of Output for Program COMBI

**** INITIALIZATION PHASE ****

PERIOD	OBSERVATION	FORECAST	ERROR	CUM. ERROR	SUM OF SQ. ERROR
2	14.0000	14.0516	-.0516	-.0516	.0027
3	14.0000	14.0082	-.0082	-.0598	.0027
4	14.0000	13.9178	.0822	.0224	.0095
5	14.1000	13.9738	.1262	.1486	.0254
6	13.7000	13.9538	-.2538	-.1052	.0898
7	13.7000	13.7696	-.0696	-.1749	.0947
8	13.7000	13.7709	-.0709	-.2458	.0997
9	13.7000	13.7665	-.0665	-.3123	.1041
10	13.6000	13.5902	.0098	-.3025	.1042
11	13.6000	13.6462	-.0462	-.3487	.1064
12	14.0000	13.5324	.4676	.1189	.3250
13	13.7000	13.8642	-.1642	-.0453	.3520
14	13.7000	13.7716	-.0716	-.1169	.3571
15	13.7000	13.7125	-.0125	-.1294	.3573
16	13.6000	13.6300	-.0300	-.1594	.3582
17	13.7000	13.6156	.0844	-.0751	.3653
18	13.7000	13.5487	.1513	.0762	.3882
19	13.7000	13.6069	.0931	.1693	.3968
20	13.6000	13.7098	-.1098	.0595	.4089
21	13.7000	13.6820	.0180	.0775	.4092
22	13.6000	13.5525	.0475	.1250	.4115
23	13.7000	13.6242	.0758	.2008	.4172
24	13.6000	13.5963	.0037	.2045	.4172
25	13.7000	13.5998	.1002	.3047	.4273
26	14.0000	13.6423	.3577	.6624	.5552
27	13.6000	13.8078	-.2078	.4546	.5984
28	13.6000	13.6001	-.0001	.4545	.5984
29	13.6000	13.5880	.0120	.4665	.5985
30	13.5000	13.4563	.0437	.5101	.6004
31	13.4000	13.4207	-.0207	.4894	.6008
32	13.5000	13.4085	.0915	.5809	.6092
33	13.5000	13.4723	.0277	.6086	.6100
34	13.1000	13.3351	-.2351	.3735	.6652
35	13.2000	13.2213	-.0213	.3522	.6657
36	12.5000	13.1131	-.6131	-.2609	1.0416
37	12.4000	12.7570	-.3570	-.6178	1.1690
38	12.6000	12.5415	.0585	-.5594	1.1724
39	12.5000	12.5037	-.0037	-.5631	1.1724
40	12.5000	12.4524	.0476	-.5155	1.1747
41	12.4000	12.5106	-.1106	-.6261	1.1869
42	12.3000	12.3868	-.0868	-.7129	1.1945
43	12.4000	12.3434	.0566	-.6563	1.1977
44	12.5000	12.4563	.0437	-.6126	1.1996
45	12.6000	12.5748	.0252	-.5873	1.2002
46	12.5000	12.5235	-.0235	-.6109	1.2008
47	12.5000	12.6364	-.1364	-.7473	1.2194
48	12.4000	12.5282	-.1282	-.8755	1.2358
49	12.6000	12.5551	.0449	-.8306	1.2378
50	12.7000	12.6837	.0163	-.8143	1.2381
51	12.7000	12.6849	.0151	-.7992	1.2383
52	12.6000	12.6962	-.0962	-.8954	1.2476
53	12.7000	12.6918	.0082	-.8872	1.2476
54	12.7000	12.6384	.0616	-.8256	1.2514
55	12.5000	12.6887	-.1887	-1.0143	1.2871
56	12.7000	12.6453	.0547	-.9596	1.2901
57	12.7000	12.7325	-.0325	-.9921	1.2911
58	12.6000	12.6031	-.0031	-.9952	1.2911
59	12.7000	12.6769	.0231	-.9721	1.2917
60	12.7000	12.6078	.0922	-.8800	1.3002
61	13.0000	12.7285	.2715	-.6085	1.3739

Continuation Example of Output for Program COMBI

62	12.6000	12.9274	-.3274	-.9359	1.4810
63	13.0000	12.6863	.3137	-.6222	1.5794
64	12.7000	12.7992	-.0992	-.7215	1.5893
65	12.6000	12.7401	-.1401	-.8616	1.6089
66	12.6000	12.5538	.0462	-.8154	1.6111
67	12.5000	12.5182	-.0182	-.8336	1.6114
68	12.5000	12.5373	-.0373	-.8709	1.6128
69	12.6000	12.5151	.0849	-.7859	1.6200
70	12.5000	12.4169	.0831	-.7028	1.6269
71	12.6000	12.4985	.1015	-.6014	1.6372
72	12.5000	12.4451	.0549	-.5464	1.6402
73	12.4000	12.5189	-.1189	-.6653	1.6543
74	12.5000	12.4285	.0715	-.5938	1.6594
75	12.4000	12.4298	-.0298	-.6236	1.6603

SUM OF FORECAST ERRORS= -.6236 AVERAGE FORECAST ERROR= -.0249 VARIANCE= .0243
MEAN SQUARE ERROR= .244166E-01

WEIGHTS UPDATED USING ALPHA= .80

Continuation Example of Output for Program COMBI

**** FORECASTING PHASE ****

PERIOD	OBSERVATION	FORECAST	ERROR	CUM. ERROR	SUM OF SQ. ERROR
76	12.4000	12.3078	.0922	.0922	.0085
77	12.5000	12.4240	.0760	.1682	.0143
78	12.6000	12.5393	.0607	.2289	.0180
79	12.6000	12.6301	-.0301	.1988	.0189
80	12.5000	12.5647	-.0647	.1341	.0231
81	12.5000	12.5134	-.0134	.1207	.0232
82	12.6000	12.4438	.1562	.2769	.0476
83	12.5000	12.5457	-.0457	.2312	.0497
84	12.7000	12.5605	.1395	.3707	.0692
85	12.5000	12.5772	-.0772	.2935	.0751
86	12.7000	12.6263	.0737	.3672	.0806
87	12.7000	12.6938	.0062	.3734	.0806
88	12.7000	12.6848	.0152	.3886	.0809
89	12.6000	12.6540	-.0540	.3346	.0838
90	12.6000	12.5711	.0289	.3635	.0846
91	12.6000	12.5635	.0365	.4000	.0859
92	12.6000	12.6188	-.0188	.3812	.0863
93	12.6000	12.6244	-.0244	.3568	.0869
94	12.7000	12.5985	.1015	.4583	.0972
95	12.7000	12.7249	-.0249	.4334	.0978
96	12.7000	12.6963	.0037	.4371	.0978
97	12.7000	12.7335	-.0335	.4035	.0989
98	12.7000	12.7304	-.0304	.3732	.0999
99	13.0000	12.8689	.1311	.5042	.1170
100	12.5000	12.6739	-.1739	.3304	.1473

SUM OF FORECAST ERRORS= .3304 AVERAGE FORECAST ERROR= .0132 VARIANCE= .0060 STANDARD DEVIATION= .0772
MEAN SQUARE ERROR= .613611E-02

WEIGHTS UPDATED USING ALPHA= .60
FORMULA 1 USES NU= 6
FORMULA 2 USES BETA= 6.0000
FORMULA 3 USE GAMMA= .8000
FORMULA 4 USES ZETA= 1.4000
FORMULA 5 USES XI= 1.2000

Continuation Example of Output for Program COMBI

**** COMBINED FORECASTS ****

PERIOD	DATA	AVERAGE	FORMULA 1	FORMULA 2	FORMULA 3	FORMULA 4	FORMULA 5	FORMULA 6(*)
76	12.4000	12.2750	12.3029	12.3017	12.2803	12.2966	12.2750	12.3078
77	12.5000	12.2950	12.3505	12.3467	12.3134	12.3359	12.3321	12.4240
78	12.6000	12.2800	12.3520	12.3374	12.3088	12.3281	12.3237	12.5393
79	12.6000	12.3250	12.5012	12.4136	12.3808	12.4044	12.3971	12.6301
80	12.5000	12.4000	12.8441	12.5866	12.5076	12.5251	12.5136	12.5647
81	12.5000	12.3750	12.6482	12.5589	12.4995	12.5025	12.4908	12.5134
82	12.6000	12.2550	12.3986	12.3727	12.3406	12.3344	12.3271	12.4438
83	12.5000	12.3500	12.6327	12.5555	12.5072	12.4847	12.4723	12.5457
84	12.7000	12.2550	12.4359	12.4171	12.3840	12.3584	12.3489	12.5605
85	12.5000	12.3650	12.6919	12.6376	12.5851	12.5310	12.5157	12.5772
86	12.7000	12.3150	12.5458	12.5535	12.5235	12.4666	12.4526	12.6263
87	12.7000	12.3300	12.6522	12.6256	12.5797	12.5033	12.4873	12.6938
88	12.7000	12.3000	12.6452	12.6031	12.5549	12.4685	12.4529	12.6848
89	12.6000	12.3300	12.7184	12.6909	12.6352	12.5227	12.5047	12.6540
90	12.6000	12.2500	12.5819	12.5686	12.5274	12.4186	12.4029	12.5711
91	12.6000	12.2150	12.5126	12.5330	12.4910	12.3765	12.3613	12.5635
92	12.6000	12.2500	12.6429	12.6329	12.5836	12.4381	12.4204	12.6188
93	12.6000	12.2600	12.6726	12.6832	12.6389	12.4676	12.4478	12.6244
94	12.7000	12.1700	12.4996	12.5228	12.4943	12.3439	12.3273	12.5985
95	12.7000	12.2450	12.6847	12.6844	12.6608	12.4652	12.4438	12.7249
96	12.7000	12.2100	12.6717	12.6502	12.6270	12.4283	12.4066	12.6963
97	12.7000	12.2450	12.7717	12.7396	12.7183	12.4906	12.4656	12.7335
98	12.7000	12.2600	12.8021	12.7899	12.7709	12.5234	12.4959	12.7304
99	13.0000	12.2150	12.7267	12.7294	12.7152	12.4719	12.4444	12.8689
100	12.5000	12.2700	12.9315	12.8490	12.8431	12.5668	12.5321	12.6739
MEAN ERROR=		.3384	.0113	.0446	.0811	.1779	.1943	.0132
ERROR VARIANCE=		.0242	.0325	.0229	.0222	.0184	.0184	.0060
STD.DEV.=		.1556	.1804	.1512	.1489	.1356	.1356	.0772
MSE ERROR=		.1435	.0327	.0249	.0290	.0513	.0577	.0061

RMIN=0. RMAX=14.1 OPTION=0.JDIV=10

Continuation Example of Output for Program COMBI

**** FORECASTING PHASE ****

PERIOD	OBSERVATION	FORECAST	ERROR	CUM. ERROR	SUM OF SQ. ERROR
76	12.4000	12.3203	.0797	.0797	.0064
77	12.5000	12.4783	.0217	.1014	.0068
78	12.6000	12.5932	.0068	.1082	.0069
79	12.6000	12.6200	-.0200	.0882	.0073
80	12.5000	12.5315	-.0315	.0568	.0083
81	12.5000	12.5012	-.0012	.0555	.0083
82	12.6000	12.4957	.1043	.1598	.0191
83	12.5000	12.5284	-.0284	.1314	.0199
84	12.7000	12.6191	.0809	.2124	.0265
85	12.5000	12.5392	-.0392	.1732	.0280
86	12.7000	12.6519	.0481	.2213	.0303
87	12.7000	12.7007	-.0007	.2206	.0303
88	12.7000	12.6957	.0043	.2249	.0304
89	12.6000	12.6265	-.0265	.1983	.0311
90	12.6000	12.5790	.0210	.2193	.0315
91	12.6000	12.5808	.0192	.2385	.0319
92	12.6000	12.6116	-.0116	.2269	.0320
93	12.6000	12.6121	-.0121	.2148	.0322
94	12.7000	12.6437	.0563	.2711	.0353
95	12.7000	12.7175	-.0175	.2536	.0356
96	12.7000	12.6983	.0017	.2553	.0356
97	12.7000	12.7190	-.0190	.2364	.0360
98	12.7000	12.7141	-.0141	.2223	.0362
99	13.0000	12.9265	.0735	.2957	.0416
100	12.5000	12.5946	-.0946	.2011	.0505

SUM OF FORECAST ERRORS= .2011 AVERAGE FORECAST ERROR= .0080 VARIANCE= .0020 STANDARD DEVIATION= .0452
MEAN SQUARE ERROR= .210594E-02

WEIGHTS UPDATED USING ALPHA= .80
FORMULA 1 USES NU= 6
FORMULA 2 USES BETA= 6.0000
FORMULA 3 USE GAMMA= .8000
FORMULA 4 USES ZETA= 1.4000
FORMULA 5 USES XI= 1.2000

Continuation Example of Output for Program COMBI

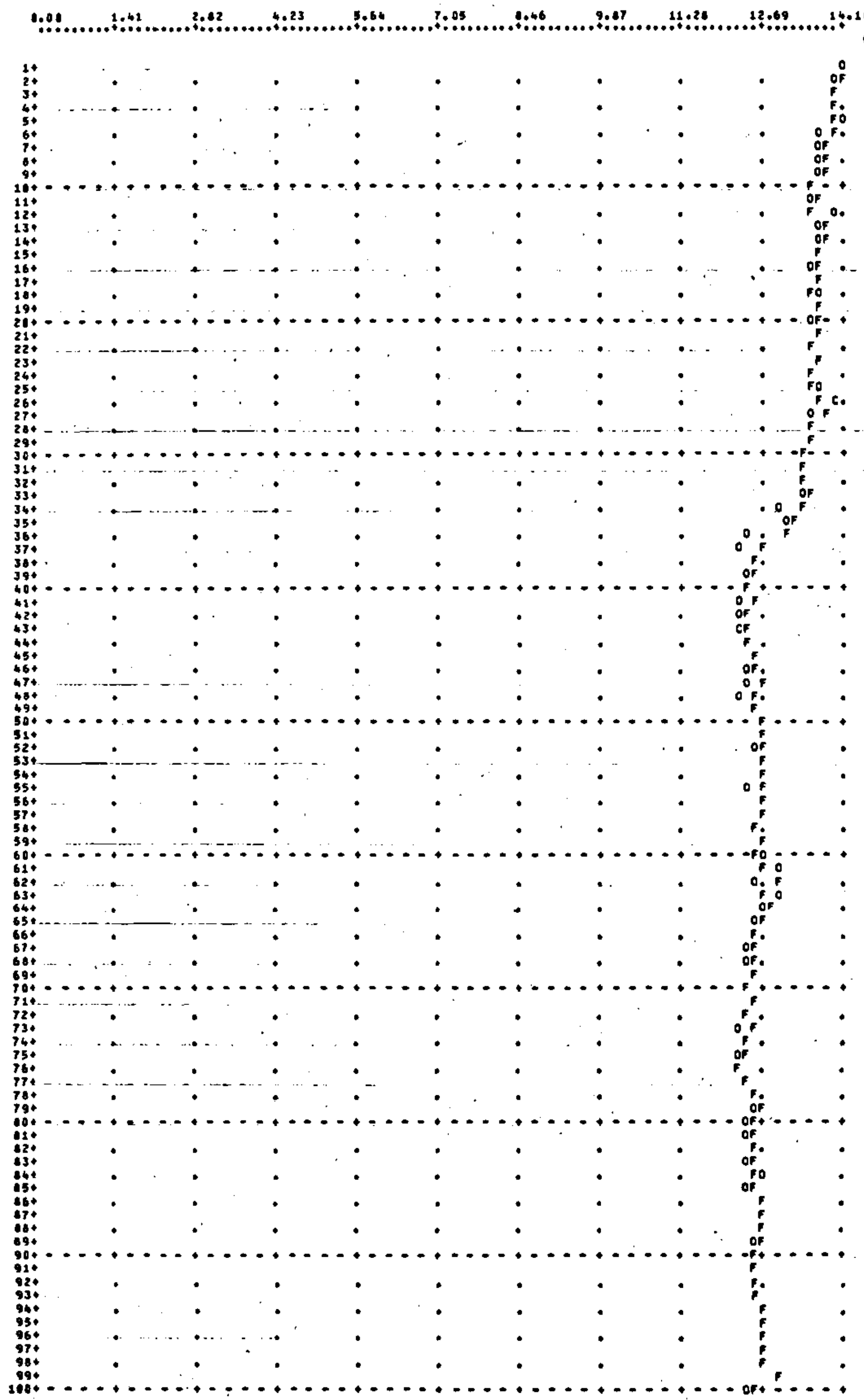
**** COMBINED FORECASTS ****

PERIOD	DATA	AVERAGE	FORMULA 1	FORMULA 2	FORMULA 3	FORMULA 4	FORMULA 5	FORMULA 6(*)
76	12.4000	12.2750	12.3029	12.3017	12.2803	12.2966	12.2750	12.3203
77	12.5000	12.2950	12.3505	12.3467	12.3134	12.3359	12.3321	12.4783
78	12.6000	12.2800	12.3520	12.3374	12.3088	12.3281	12.3237	12.5932
79	12.6000	12.3250	12.5012	12.4136	12.3808	12.4044	12.3971	12.6200
80	12.5000	12.4000	12.8441	12.5866	12.5076	12.5251	12.5136	12.5315
81	12.5000	12.3750	12.6482	12.5589	12.4995	12.5025	12.4908	12.5012
82	12.6000	12.2550	12.3986	12.3727	12.3406	12.3344	12.3271	12.4957
83	12.5000	12.3500	12.6327	12.5555	12.5072	12.4847	12.4723	12.5284
84	12.7000	12.2550	12.4359	12.4171	12.3840	12.3584	12.3489	12.6191
85	12.5000	12.3650	12.6919	12.6376	12.5851	12.5310	12.5157	12.5392
86	12.7000	12.3150	12.5458	12.5535	12.5235	12.4666	12.4526	12.6519
87	12.7000	12.3300	12.6522	12.6256	12.5797	12.5033	12.4873	12.7007
88	12.7000	12.3000	12.6452	12.6031	12.5549	12.4685	12.4529	12.6957
89	12.6000	12.3300	12.7184	12.6909	12.6352	12.5227	12.5047	12.6265
90	12.6000	12.2500	12.5819	12.5686	12.5274	12.4186	12.4029	12.5790
91	12.6000	12.2150	12.5126	12.5330	12.4910	12.3765	12.3613	12.5808
92	12.6000	12.2500	12.6429	12.6329	12.5836	12.4381	12.4204	12.6116
93	12.6000	12.2600	12.6726	12.6832	12.6389	12.4676	12.4478	12.6121
94	12.7000	12.1700	12.4996	12.5228	12.4943	12.3439	12.3273	12.6437
95	12.7000	12.2450	12.6847	12.6844	12.6608	12.4652	12.4438	12.7175
96	12.7000	12.2100	12.6717	12.6502	12.6270	12.4283	12.4066	12.6983
97	12.7000	12.2450	12.7717	12.7396	12.7183	12.4906	12.4656	12.7190
98	12.7000	12.2600	12.8021	12.7899	12.7709	12.5234	12.4959	12.7141
99	13.0000	12.2150	12.7267	12.7294	12.7152	12.4719	12.4444	12.9265
100	12.5000	12.2700	12.9315	12.8490	12.8431	12.5668	12.5321	12.5946
MEAN ERROR=		.3384	.0113	.0446	.0811	.1779	.1943	.0080
ERROR VARIANCE=		.0242	.0325	.0229	.0222	.0184	.0184	.0020
STD.DEV.=		.1556	.1804	.1512	.1489	.1356	.1356	.0452
MSE ERROR=		.1435	.0327	.0249	.0290	.0513	.0577	.0021

RMIN=0. RMAX=14.1 OPTION=0.JOIV=10

Continuation Example of Output for Program COMBI

PLOT OF OBSERVED & FORECASTED TIME SERIES



	OBSERV	FORCST
1	14.10	0.00
2	14.00	14.05
3	14.00	14.01
4	14.00	13.92
5	14.10	13.97
6	13.70	13.95
7	13.70	13.77
8	13.70	13.77
9	13.70	13.77
10	13.60	13.59
11	13.60	13.65
12	14.00	13.53
13	13.70	13.86
14	13.70	13.77
15	13.70	13.71
16	13.60	13.63
17	13.70	13.62
18	13.70	13.55
19	13.70	13.61
20	13.60	13.71
21	13.70	13.68
22	13.60	13.55
23	13.70	13.62
24	13.60	13.60
25	13.70	13.60
26	14.00	13.64
27	13.60	13.81
28	13.60	13.60
29	13.60	13.59
30	13.50	13.46
31	13.40	13.42
32	13.50	13.41
33	13.50	13.47
34	13.10	13.34
35	13.20	13.22
36	12.50	13.11
37	12.40	12.76
38	12.60	12.54
39	12.50	12.50
40	12.50	12.45
41	12.40	12.51
42	12.30	12.39
43	12.40	12.34
44	12.50	12.46
45	12.60	12.57
46	12.50	12.52
47	12.50	12.64
48	12.40	12.53
49	12.60	12.56
50	12.70	12.68
51	12.70	12.68
52	12.60	12.70
53	12.70	12.69
54	12.70	12.64
55	12.50	12.69
56	12.70	12.65
57	12.70	12.73
58	12.60	12.60
59	12.70	12.68
60	12.70	12.61
61	13.00	12.73
62	12.60	12.93
63	13.00	12.69
64	12.70	12.80
65	12.60	12.74
66	12.60	12.55
67	12.50	12.52
68	12.50	12.54
69	12.60	12.52
70	12.50	12.42
71	12.60	12.50
72	12.50	12.45
73	12.40	12.52
74	12.50	12.43
75	12.40	12.43
76	12.40	12.32
77	12.50	12.48
78	12.60	12.59
79	12.60	12.62
80	12.50	12.53
81	12.50	12.50
82	12.60	12.58
83	12.50	12.53
84	12.70	12.62
85	12.50	12.54
86	12.70	12.65
87	12.70	12.70
88	12.70	12.70
89	12.60	12.63
90	12.60	12.58
91	12.60	12.58
92	12.60	12.61
93	12.60	12.61
94	12.70	12.64
95	12.70	12.72
96	12.70	12.70
97	12.70	12.72
98	12.70	12.71
99	13.00	12.93
100	12.50	12.59

Appendix 0.1 Listing of data points for Series 1*

INTERNATIONAL AIRLINE PASSENGERS-MONTHLY TOTALS JAN 1949-DEC 1960 THOUSANDS

112.0	118.0	132.0	129.0	121.0	135.0	148.0	148.0
136.0	119.0	104.0	118.0	115.0	126.0	141.0	135.0
125.0	149.0	170.0	170.0	158.0	133.0	114.0	140.0
145.0	150.0	178.0	163.0	172.0	178.0	199.0	199.0
184.0	162.0	146.0	166.0	171.0	180.0	193.0	181.0
183.0	218.0	230.0	242.0	209.0	191.0	172.0	194.0
196.0	196.0	236.0	235.0	229.0	243.0	264.0	272.0
237.0	211.0	180.0	201.0	204.0	188.0	235.0	227.0
234.0	264.0	302.0	293.0	259.0	229.0	203.0	229.0
242.0	233.0	267.0	269.0	270.0	315.0	364.0	347.0
312.0	274.0	237.0	278.0	284.0	277.0	317.0	313.0
318.0	374.0	413.0	405.0	355.0	306.0	271.0	306.0
315.0	301.0	356.0	348.0	355.0	422.0	465.0	467.0
404.0	347.0	305.0	336.0	340.0	318.0	362.0	348.0
363.0	435.0	491.0	505.0	404.0	359.0	310.0	337.0
360.0	342.0	406.0	396.0	420.0	472.0	548.0	559.0
463.0	407.0	362.0	405.0	417.0	391.0	419.0	461.0
472.0	535.0	622.0	606.0	508.0	461.0	390.0	432.0

* Reference [9] pp.531

Appendix D.2 Listing of data points for Series 2*

IBM COMMON STOCK CLOSING PRICES- DAILY, 17TH MAY 1961-2ND NOV 1962

460.0	457.0	452.0	459.0	462.0	459.0	463.0	479.0
493.0	490.0	492.0	498.0	499.0	497.0	496.0	490.0
489.0	478.0	487.0	491.0	487.0	482.0	479.0	478.0
479.0	477.0	479.0	475.0	479.0	476.0	476.0	478.0
479.0	477.0	476.0	475.0	475.0	473.0	474.0	474.0
474.0	465.0	466.0	467.0	471.0	471.0	467.0	473.0
481.0	488.0	490.0	489.0	489.0	485.0	491.0	492.0
494.0	499.0	498.0	500.0	497.0	494.0	495.0	500.0
504.0	513.0	511.0	514.0	510.0	509.0	515.0	519.0
523.0	519.0	523.0	531.0	547.0	551.0	547.0	541.0
545.0	549.0	545.0	549.0	547.0	543.0	540.0	539.0
532.0	517.0	527.0	540.0	542.0	538.0	541.0	541.0
547.0	553.0	559.0	557.0	557.0	560.0	571.0	571.0
569.0	575.0	580.0	584.0	585.0	590.0	599.0	603.0
599.0	596.0	585.0	587.0	585.0	581.0	583.0	592.0
592.0	596.0	596.0	595.0	598.0	598.0	595.0	595.0
592.0	588.0	582.0	576.0	578.0	589.0	585.0	580.0
579.0	584.0	581.0	581.0	577.0	577.0	578.0	580.0
586.0	583.0	581.0	576.0	571.0	575.0	575.0	573.0
577.0	582.0	584.0	579.0	572.0	577.0	571.0	560.0
549.0	556.0	557.0	563.0	564.0	567.0	561.0	559.0
553.0	553.0	553.0	547.0	550.0	544.0	541.0	532.0
525.0	542.0	555.0	558.0	551.0	551.0	552.0	553.0
557.0	557.0	548.0	547.0	545.0	545.0	539.0	539.0
535.0	537.0	535.0	536.0	537.0	543.0	548.0	546.0
547.0	548.0	549.0	553.0	553.0	552.0	551.0	550.0
553.0	554.0	551.0	551.0	545.0	547.0	547.0	537.0
539.0	538.0	533.0	525.0	513.0	510.0	521.0	521.0
521.0	523.0	516.0	511.0	518.0	517.0	520.0	519.0
519.0	519.0	518.0	513.0	499.0	485.0	454.0	462.0
473.0	482.0	486.0	475.0	459.0	451.0	453.0	446.0
455.0	452.0	457.0	449.0	450.0	435.0	415.0	398.0
399.0	361.0	383.0	393.0	385.0	360.0	364.0	365.0
370.0	374.0	359.0	335.0	323.0	306.0	333.0	330.0
336.0	328.0	316.0	320.0	332.0	320.0	333.0	344.0
339.0	350.0	351.0	350.0	345.0	350.0	359.0	375.0
379.0	376.0	382.0	370.0	365.0	367.0	372.0	373.0
363.0	371.0	369.0	376.0	387.0	387.0	376.0	385.0
385.0	380.0	373.0	382.0	377.0	376.0	379.0	386.0
387.0	386.0	389.0	394.0	393.0	409.0	411.0	409.0
408.0	393.0	391.0	388.0	396.0	387.0	383.0	388.0
382.0	384.0	382.0	383.0	383.0	388.0	395.0	392.0
386.0	383.0	377.0	364.0	369.0	355.0	350.0	353.0
340.0	350.0	349.0	358.0	360.0	360.0	366.0	359.0
356.0	355.0	367.0	357.0	361.0	355.0	348.0	343.0
330.0	340.0	339.0	331.0	345.0	352.0	346.0	352.0
357.0							

* Reference [9] pp.526

Appendix D.3 Listing of data points for Series 3*

IBM COMMON STOCK CLOSING PRICES-DAILY, 29TH JUNE 1959- 30TH JUNE 1960

445.0	448.0	450.0	447.0	451.0	453.0	454.0	454.0
459.0	440.0	446.0	443.0	443.0	440.0	439.0	435.0
435.0	436.0	435.0	435.0	435.0	433.0	429.0	428.0
425.0	427.0	425.0	422.0	409.0	407.0	423.0	422.0
417.0	421.0	424.0	414.0	419.0	429.0	426.0	425.0
424.0	425.0	425.0	424.0	425.0	421.0	414.0	410.0
411.0	406.0	406.0	413.0	411.0	410.0	405.0	409.0
410.0	405.0	401.0	401.0	401.0	414.0	419.0	425.0
423.0	411.0	414.0	420.0	412.0	415.0	412.0	412.0
411.0	412.0	409.0	407.0	408.0	415.0	413.0	413.0
410.0	405.0	410.0	412.0	413.0	411.0	411.0	409.0
406.0	407.0	410.0	408.0	408.0	409.0	410.0	409.0
405.0	406.0	405.0	407.0	409.0	407.0	409.0	425.0
425.0	428.0	436.0	442.0	442.0	433.0	435.0	433.0
435.0	429.0	439.0	437.0	439.0	438.0	435.0	433.0
437.0	437.0	444.0	441.0	440.0	441.0	439.0	439.0
438.0	437.0	441.0	442.0	441.0	437.0	427.0	423.0
424.0	428.0	428.0	431.0	425.0	423.0	420.0	426.0
418.0	416.0	419.0	418.0	416.0	419.0	425.0	421.0
422.0	422.0	417.0	420.0	417.0	418.0	419.0	419.0
417.0	419.0	422.0	423.0	422.0	421.0	421.0	419.0
418.0	421.0	420.0	413.0	413.0	408.0	409.0	415.0
415.0	420.0	420.0	424.0	426.0	423.0	423.0	425.0
431.0	436.0	436.0	440.0	436.0	443.0	445.0	439.0
443.0	445.0	450.0	461.0	471.0	467.0	462.0	456.0
464.0	463.0	465.0	464.0	456.0	460.0	458.0	453.0
453.0	449.0	447.0	453.0	450.0	459.0	457.0	453.0
455.0	453.0	450.0	456.0	461.0	463.0	463.0	461.0
465.0	473.0	473.0	475.0	499.0	485.0	491.0	496.0
504.0	504.0	509.0	511.0	524.0	525.0	541.0	531.0
529.0	530.0	531.0	527.0	525.0	519.0	514.0	509.0
505.0	513.0	525.0	519.0	519.0	522.0	522.0	

* Reference [9] pp.527

Appendix D.4 Listing of data points for Series 4 *

CHEMICAL PROCESS VISCOSITY READINGS-EVERY HOUR

8.0	8.0	7.4	8.0	8.0	8.0	8.0	8.8
8.4	8.4	8.0	8.2	8.2	8.2	8.4	8.4
8.4	8.6	8.8	8.6	8.6	8.6	8.6	8.6
8.8	8.9	9.1	9.5	8.5	8.4	8.3	8.2
8.1	8.3	8.4	8.7	8.8	8.8	9.2	9.6
9.0	8.8	8.6	8.6	8.8	8.8	8.6	8.6
8.4	8.3	8.4	8.3	8.3	8.1	8.2	8.3
8.5	8.1	8.1	7.9	8.3	8.1	8.1	8.1
8.4	8.7	9.0	9.3	9.3	9.5	9.3	9.5
9.5	9.5	9.5	9.5	9.5	9.9	9.5	9.7
9.1	9.1	8.9	9.3	9.1	9.1	9.3	9.5
9.3	9.3	9.3	9.9	9.7	9.1	9.3	9.5
9.4	9.0	9.0	8.8	9.0	8.8	8.6	8.6
8.0	8.0	8.0	8.0	8.6	8.0	8.0	8.0
7.6	8.6	9.6	9.6	10.0	9.4	9.3	9.2
9.5	9.5	9.5	9.9	9.9	9.5	9.3	9.5
9.5	9.1	9.3	9.5	9.3	9.1	9.3	9.1
9.5	9.4	9.5	9.6	10.2	9.8	9.6	9.6
9.4	9.4	9.4	9.4	9.6	9.6	9.4	9.4
9.0	9.4	9.4	9.6	9.4	9.2	8.8	8.8
9.2	9.2	9.6	9.6	9.8	9.8	10.0	10.0
9.4	9.8	8.8	8.8	8.8	8.8	9.6	9.6
9.6	9.2	9.2	9.0	9.0	9.0	9.4	9.0
9.0	9.4	9.4	9.6	9.4	9.6	9.6	9.6
10.0	10.0	9.6	9.2	9.2	9.2	9.0	9.0
9.6	9.8	10.2	10.0	10.0	10.0	9.4	9.2
9.6	9.7	9.7	9.8	9.8	9.8	10.0	10.0
8.6	9.0	9.4	9.4	9.4	9.4	9.4	9.6
10.0	10.0	9.8	9.8	9.7	9.6	9.4	9.2
9.0	9.4	9.6	9.6	9.6	9.6	9.6	9.6
9.0	9.4	9.4	9.4	9.6	9.4	9.6	9.6
9.8	9.8	9.8	9.6	9.2	9.6	9.2	9.2
9.6	9.6	9.6	9.6	9.6	9.6	10.0	10.0
10.4	10.4	9.8	9.0	9.6	9.8	9.6	8.6
8.0	8.0	8.0	8.0	8.4	8.8	8.4	8.4
9.0	9.0	9.4	10.0	10.0	10.0	10.2	10.0
10.0	9.6	9.0	9.0	8.6	9.0	9.6	9.6
9.0	9.0	8.9	8.8	8.7	8.6	8.3	7.9
8.5	8.7	8.9	9.1	9.1	9.1		

* Reference [9] pp.529

Appendix D.5 Listing of data points for Series 5*

US AUTO REGISTRATIONS

209.0	214.0	265.0	290.0	287.0	270.0	263.0	265.0
252.0	281.0	259.0	312.0	275.0	250.0	312.0	331.0
256.0	247.0	291.0	318.0	296.0	291.0	313.0	311.0
273.0	258.0	361.0	391.0	446.0	433.0	449.0	479.0
460.0	466.0	410.0	415.0	382.0	409.0	496.0	471.0
488.0	584.0	610.0	684.0	626.0	580.0	444.0	552.0
473.0	431.0	513.0	467.0	470.0	455.0	406.0	424.0
406.0	373.0	332.0	310.0	301.0	296.0	333.0	374.0
422.0	424.0	341.0	216.0	319.0	383.0	360.0	400.0
386.0	397.0	486.0	528.0	541.0	542.0	534.0	502.0
454.0	505.0	450.0	414.0	341.0	370.0	481.0	508.0
521.0	597.0	474.0	440.0	408.0	396.0	381.0	657.0
440.0	477.0	637.0	652.0	661.0	681.0	647.0	659.0
655.0	576.0	509.0	631.0	432.0	448.0	545.0	564.0
560.0	540.0	535.0	568.0	421.0	424.0	404.0	514.0
437.0	439.0	573.0	549.0	556.0	517.0	543.0	492.0
495.0	464.0	409.0	512.0	382.0	334.0	401.0	418.0
424.0	411.0	400.0	371.0	317.0	321.0	335.0	511.0
421.0	425.0	498.0	575.0	584.0	586.0	567.0	534.0
458.0	535.0	429.0	431.0	430.0	494.0	597.0	647.0
647.0	596.0	547.0	525.0	459.0	548.0	543.0	544.0
414.0	375.0	480.0	496.0	544.0	572.0	501.0	471.0
371.0	550.0	558.0	526.0	506.0	473.0	592.0	635.0
644.0	602.0	614.0	540.0	374.0	678.0	638.0	644.0
554.0	498.0	624.0	759.0	715.0	692.0	706.0	553.0
404.0	715.0	640.0	712.0	612.0	552.0	637.0	812.0
781.0	754.0	724.0	649.0	565.0	659.0	564.0	757.0
667.0	631.0	799.0	896.0	841.0	842.0	834.0	767.0
590.0	746.0	794.0	909.0	607.0	722.0	879.0	823.0
777.0	753.0	833.0	744.0	574.0	767.0	732.0	808.0
616.0	539.0	671.0	786.0	822.0	806.0	753.0	726.0
550.0	710.0	643.0	738.0	658.0	605.0	725.0	859.0
824.0	801.0	872.0	744.0	705.0	880.0	757.0	977.0

* Reference: Nelson, C.R. Applied Time Series Analysis for Managerial Forecasting. Holden Day, San Francisco, 1973.

Appendix D.6 Listing of data points for Series 6*

CHEMICAL PROCESS TEMPERATURE READINGS, EVERY TWO MINUTES

153.0	145.0	142.0	145.0	175.0	170.0	159.0	173.0
140.0	136.0	185.0	173.0	155.0	150.0	148.0	157.0
140.0	133.0	165.0	178.0	149.0	141.0	166.0	166.0
134.0	137.0	146.0	152.0	135.0	141.0	137.0	151.0
182.0	148.0	129.0	138.0	149.0	161.0	144.0	145.0
150.0	143.0	156.0	159.0	169.0	174.0	141.0	133.0
160.0	168.0	159.0	156.0	159.0	139.0	148.0	158.0
168.0	173.0	149.0	142.0	165.0	167.0	162.0	148.0
137.0	155.0	137.0	140.0	162.0	163.0	147.0	150.0
139.0	158.0	153.0	156.0	158.0	155.0	147.0	141.0
138.0	145.0	162.0	167.0	149.0	143.0	152.0	133.0
128.0	138.0	157.0	170.0	133.0	135.0	170.0	160.0
120.0	134.0	128.0	138.0				

* Reference [1]- pp. 272

Appendix D.7 Listing of data points for Series 7*

YEARLY WOLFER SUNSPOT NUMBERS %AVE. NUMBER OF SUNSPOTS/YEAR

101.0	82.0	66.0	35.0	31.0	7.0	20.0	92.0
154.0	125.0	85.0	68.0	38.0	23.0	10.0	24.0
83.0	132.0	131.0	118.0	90.0	67.0	60.0	47.0
41.0	21.0	16.0	6.0	4.0	7.0	14.0	34.0
45.0	43.0	48.0	42.0	28.0	10.0	8.0	2.0
0.0	1.0	5.0	12.0	14.0	35.0	46.0	41.0
30.0	24.0	16.0	7.0	4.0	2.0	8.0	17.0
36.0	50.0	62.0	67.0	71.0	48.0	28.0	8.0
13.0	57.0	122.0	138.0	103.0	86.0	63.0	37.0
24.0	11.0	15.0	40.0	62.0	98.0	124.0	96.0
66.0	64.0	54.0	39.0	21.0	7.0	4.0	23.0
55.0	94.0	96.0	77.0	59.0	44.0	47.0	30.0
16.0	7.0	37.0	74.0				

* Reference [1]- pp. 274

Appendix D.8 Listing of data points for Series 8*

MINUTES OF USAGE PER DAY FOR A COMPUTER

160.0	158.0	150.0	151.0	150.0	151.0	153.0	157.0
156.0	158.0	162.0	161.0	162.0	163.0	161.0	160.0
158.0	159.0	157.0	159.0	160.0	162.0	170.0	172.0
177.0	184.0	186.0	187.0	195.0	202.0	203.0	205.0
208.0	209.0	214.0	215.0	209.0	203.0	199.0	192.0
193.0	190.0	189.0	185.0	182.0	181.0	180.0	181.0
184.0	186.0	187.0	185.0	188.0	192.0	198.0	197.0
193.0	190.0	183.0	178.0	180.0	183.0	187.0	193.0
196.0	197.0	198.0	201.0	200.0	201.0	204.0	206.0
210.0	211.0	216.0	212.0	211.0	210.0	208.0	205.0
202.0	201.0	203.0	205.0	197.0	188.0	186.0	185.0
184.0	183.0	187.0	173.0	171.0	173.0	172.0	174.0
175.0	171.0	172.0	175.0				

* Reference [1]- pp. 270

Appendix D.9 Listing of data points for Series 9*

CHEMICAL PROCESS TEMPERATURE READINGS - EVERY MINUTE

26.6	27.0	27.1	27.1	27.1	27.1	26.9	26.8
26.7	26.4	26.0	25.8	25.6	25.2	25.0	24.6
24.2	24.0	23.7	23.4	23.1	22.9	22.8	22.7
22.6	22.4	22.2	22.0	21.8	21.4	20.9	20.3
19.7	19.4	19.3	19.2	19.1	19.0	18.9	18.9
19.2	19.3	19.3	19.4	19.5	19.6	19.6	19.6
19.6	19.6	19.7	19.9	20.0	20.1	20.2	20.3
20.6	21.6	21.9	21.7	21.3	21.2	21.4	21.7
22.2	23.0	23.8	24.6	25.1	25.6	25.8	26.1
26.3	26.3	26.2	26.0	25.8	25.6	25.4	25.2
24.9	24.7	24.5	24.4	24.4	24.4	24.4	24.4
24.3	24.4	24.4	24.4	24.4	24.4	24.5	24.5
24.4	24.3	24.2	24.2	24.0	23.9	23.7	23.6
23.5	23.5	23.5	23.5	23.5	23.7	23.8	23.8
23.9	23.9	23.8	23.7	23.6	23.4	23.2	23.0
22.8	22.6	22.4	22.0	21.6	21.3	21.2	21.2
21.1	21.0	20.9	21.0	21.0	21.1	21.2	21.1
20.9	20.8	20.8	20.8	20.8	20.9	20.8	20.8
20.7	20.7	20.8	20.9	21.2	21.4	21.7	21.8
21.9	22.2	22.5	22.8	23.1	23.4	23.8	24.1
24.6	24.9	24.9	25.1	25.0	25.0	25.0	25.0
24.9	24.8	24.7	24.6	24.5	24.5	24.5	24.5
24.5	24.5	24.5	24.4	24.4	24.2	24.2	24.1
24.1	24.0	24.0	24.0	23.9	23.8	23.8	23.7
23.7	23.6	23.7	23.6	23.6	23.6	23.5	23.5
23.4	23.3	23.3	23.3	23.4	23.4	23.3	23.2
23.3	23.3	23.2	23.1	22.9	22.8	22.6	22.4
22.2	21.8	21.3	20.8	20.2	19.7	19.3	19.1
19.0	18.8						

* Reference [9] pp.528

Appendix D.10 Listing of data points for Series 10*

DEMAND FOR A DOUBLE KNIT POLYESTER FABRIC

656.	659.	601.	624.	545.	502.	565.	577.
549.	624.	521.	520.	594.	620.	537.	640.
513.	639.	600.	617.	636.	603.	497.	600.
561.	556.	505.	704.	641.	632.	644.	677.
574.	624.	529.	512.	576.	547.	561.	592.
481.	574.	490.	614.	529.	624.	506.	617.
588.	609.	632.	550.	505.	625.	549.	600.
549.	684.	613.	648.	652.	669.	588.	624.
513.	536.	585.	552.	597.	612.	569.	496.
513.	584.	537.	656.	478.	629.	592.	601.
582.	514.	569.	612.	545.	608.	581.	700.
625.	636.	632.	657.	546.	597.	497.	544.
577.	552.	593.	628.	594.	540.	526.	576.
555.	651.	504.	609.	612.	625.	517.	556.
577.	600.	577.	616.	609.	684.	634.	628.
627.	597.	564.	604.	552.	560.	593.	576.
609.	624.	579.	507.	543.	564.	552.	645.
600.	569.	630.	625.	563.	568.	577.	624.
585.	598.	613.	687.	689.	600.	604.	481.
513.	635.	622.	576.	539.	620.	613.	648.
520.	561.	606.	609.	619.	625.	614.	614.
640.	616.	563.	608.	561.	624.	583.	560.
632.	640.	698.	592.	603.	481.	492.	629.
585.	575.	531.	640.	641.	636.	535.	565.
620.	645.	628.	637.	592.	641.	633.	615.
577.	632.	583.	637.	570.	505.	632.	648.
697.	604.	624.	467.	622.	613.	528.	568.
590.	600.	644.	640.	506.	577.	606.	621.
614.	581.	614.	627.	640.	602.	597.	636.
572.	625.	556.	488.	642.	668.	677.	637.

* Reference [1]- pp. 269

Appendix D.11 Listing of data points for Series 11*

US TREASURY BILLS INTEREST RATE, MONTHLY JAN 1956 - JAN 1969

245.6	237.2	231.0	261.3	265.0	252.7	233.4	260.6
285.0	296.1	300.0	323.0	321.0	316.5	314.0	311.3
304.2	331.6	316.5	340.4	357.8	359.1	333.7	310.2
259.8	156.2	135.4	112.6	104.6	88.1	96.2	168.6
248.4	279.3	275.6	281.4	283.7	271.2	285.2	296.0
285.1	324.7	324.3	335.8	399.8	411.7	420.9	457.2
443.6	395.4	343.9	324.4	339.2	264.1	239.6	228.6
248.9	242.6	238.4	227.2	230.2	240.8	242.0	232.7
228.8	235.9	226.8	240.2	230.4	235.0	245.8	261.7
274.6	275.2	271.9	273.5	269.4	271.9	294.5	283.7
279.2	275.1	280.3	285.6	291.4	291.6	289.7	290.9
292.0	299.5	314.3	332.0	337.9	345.3	352.2	352.3
352.9	353.2	355.3	348.4	348.2	347.8	347.9	350.6
352.7	357.5	352.4	385.6	382.8	392.9	394.2	393.2
389.5	381.0	383.1	383.6	391.2	403.2	408.2	436.2
459.6	467.0	462.6	461.1	464.2	453.9	485.5	493.2
535.6	538.7	534.4	500.7	475.9	455.4	428.8	385.2
364.0	348.0	430.8	427.5	445.1	458.8	476.2	501.2
508.1	496.9	514.4	536.5	562.1	554.4	538.2	509.5
520.2	533.4	549.2	591.6	617.7			

* Reference - Computer Programs for the Analysis of Univariate Time Series Using the Methods of Box and Jenkins- The University of Wisconsin Computer Center, Series 517

Appendix D.12 Listing of data points for Series 12*

MONTHLY VALUE OF RESIDENTIAL CONSTRUCTION JAN 1959 - DEC 1969

162.6	142.3	167.6	190.7	209.1	225.3	235.0	235.8
231.0	223.0	209.2	138.3	158.8	140.8	161.3	130.4
193.5	211.2	303.9	198.2	193.1	186.0	179.0	164.4
137.8	122.1	145.9	172.0	185.0	205.9	205.3	205.3
205.5	204.1	197.4	180.7	154.3	136.8	160.5	190.6
214.1	237.7	235.7	237.7	233.0	221.0	211.3	196.5
175.8	154.0	161.0	189.2	220.3	241.8	251.7	255.3
251.8	250.0	245.0	233.0	186.7	168.7	196.5	206.8
219.1	238.5	251.8	254.1	243.9	232.7	226.0	211.8
183.4	163.3	178.2	200.0	220.3	242.9	179.8	200.8
247.3	237.7	228.3	213.6	186.1	165.9	179.8	200.8
215.8	215.6	229.0	232.3	226.4	213.3	194.8	177.0
150.5	138.1	126.3	142.2	164.2	186.8	210.7	233.6
247.1	244.6	237.5	234.0	219.1	196.9	155.5	188.5
226.2	251.8	252.8	272.1	279.0	278.0	267.8	259.3
245.4	213.1	194.0	219.5	253.3	291.0	296.2	296.9
287.5	277.3	269.7	256.2				

* Reference - Computer Programs for the Analysis of Univariate Time Series Using the Methods of Box and Jenkins- The University of Wisconsin Computer Center, Series 517

Appendix D.13 Listing of data points for Series 13

US-DOCUMENTED MERCHANT VESSELS-TRADE-SAILING 1789-1970

20.0	47.0	50.0	56.0	52.0	62.0	75.0	83.0
88.0	89.0	94.0	97.0	95.0	89.0	95.0	104.0
114.0	120.0	127.0	124.0	135.0	142.0	123.0	127.0
116.0	115.0	136.0	136.0	139.0	121.0	124.0	126.0
127.0	130.0	131.0	137.0	140.0	150.0	158.0	170.0
121.0	113.0	119.0	135.0	150.0	163.0	170.0	173.0
174.0	180.0	190.0	198.0	196.0	186.0	192.0	200.0
209.0	221.0	243.0	272.0	287.0	301.0	319.0	349.0
380.0	413.0	444.0	420.0	423.0	432.0	437.0	448.0
466.0	440.0	458.0	401.0	403.0	323.0	311.0	251.0
240.0	236.0	228.0	232.0	238.0	247.0	258.0	261.0
258.0	252.0	242.0	237.0	235.0	236.0	238.0	241.0
237.0	221.0	217.0	212.0	209.0	212.0	217.0	218.0
212.0	203.0	196.0	192.0	190.0	184.0	182.0	188.0
193.0	194.0	197.0	194.0	196.0	189.0	181.0	176.0
171.0	165.0	159.0	154.0	151.0	143.0	138.0	131.0
128.0	121.0	120.0	127.0	129.0	129.0	125.0	118.0
112.0	109.0	98.0	91.0	82.0	76.0	67.0	63.0
56.0	50.0	44.0	38.0	31.0	26.0	22.0	20.0
18.0	17.0	14.0	13.0	12.0	9.0	9.0	9.0
9.0	8.0	7.0	6.0	5.0	5.0	4.0	3.0
2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
1.0	1.0	1.0	1.0	1.0	1.0		

* Reference: Historical Statistics of the United States Colonial Times to 1970- Part 2, U>S> Department of Commerce, Washington, D>C> 1975 pp. 749

Appendix D.14 Listing of data points for Series 14*

WEEKLY SALES OF A CUTTING TOOL

100.0	94.0	90.0	96.0	91.0	94.0	95.0	99.0
95.0	98.0	100.0	97.0	99.0	98.0	96.0	98.0
96.0	99.0	97.0	98.0	101.0	100.0	102.0	110.0
104.0	110.0	113.0	112.0	113.0	120.0	121.0	118.0
123.0	124.0	123.0	129.0	125.0	119.0	118.0	120.0
115.0	122.0	118.0	120.0	116.0	118.0	119.0	118.0
120.0	123.0	124.0	123.0	121.0	126.0	128.0	124.0
122.0	123.0	119.0	120.0	125.0	126.0	129.0	132.0
131.0	130.0	133.0	134.0	133.0	135.0	138.0	137.0
140.0	138.0	144.0	135.0	138.0	137.0	136.0	136.0
137.0	138.0	140.0	139.0	131.0	129.0	134.0	133.0
133.0	134.0	132.0	126.0	130.0	131.0	130.0	133.0
132.0	129.0	135.0	136.0				

* Reference [1]- pp. 270

Appendix D.15 Listing of data points for Series 15

DOW JONES TRANSPORTATION

224.2	223.7	224.3	225.9	225.2	224.9	224.0	223.7
222.9	221.9	221.8	221.9	223.8	223.4	224.6	224.1
223.6	223.6	222.6	223.6	224.5	225.4	227.4	228.4
229.9	231.4	230.2	229.6	228.7	228.2	226.3	225.3
226.2	223.6	223.0	206.9	207.0	210.3	210.6	208.0
206.0	203.3	200.9	201.9	201.5	204.2	205.0	206.5
211.5	212.9	213.5	212.1	212.5	210.5	211.6	212.7
212.8	210.5	212.0	211.6	212.4	214.2	219.6	220.9
220.3	219.1	219.5	217.8	216.9	217.7	219.8	218.8
214.7	212.0	212.0	211.4	213.0	213.0	213.4	214.0
213.3	214.4	212.2	211.7	210.8	211.9	214.8	217.5
215.7	217.4	219.0	220.5	221.6	221.1	220.6	221.8
222.2	221.4	222.5	224.8				

Appendix D.16 Listing of data points for Series 16

DELTA AIRLINES

14.1	14.0	14.0	14.0	14.1	13.7	13.7	13.7
13.7	13.6	13.6	14.0	13.7	13.7	13.7	13.6
13.7	13.7	13.7	13.6	13.7	13.6	13.7	13.6
13.7	14.0	13.6	13.6	13.6	13.5	13.4	13.5
13.5	13.1	13.2	12.5	12.4	12.6	12.5	12.5
12.4	12.3	12.4	12.5	12.6	12.5	12.5	12.4
12.6	12.7	12.7	12.6	12.7	12.7	12.5	12.7
12.7	12.6	12.7	12.7	13.0	12.6	13.0	12.7
12.6	12.6	12.5	12.5	12.6	12.5	12.6	12.5
12.4	12.5	12.4	12.4	12.5	12.6	12.6	12.5
12.5	12.6	12.5	12.7	12.5	12.7	12.7	12.7
12.6	12.6	12.6	12.6	12.6	12.7	12.7	12.7
12.7	12.7	13.0	12.5				

Appendix D.17 Listing of data points for Series 17

NATIONAL AIRLINES

12.0	11.7	11.7	11.6	11.6	11.3	11.0	12.2
11.0	10.4	10.5	11.0	11.3	11.5	11.7	11.6
11.7	11.7	12.2	12.1	12.1	11.7	12.0	12.0
11.6	11.5	11.2	11.1	11.0	11.1	11.0	10.6
11.1	10.7	10.6	16.0	16.0	16.4	17.1	17.0
16.0	15.4	15.4	15.5	15.4	15.5	15.6	15.7
15.5	15.5	15.4	15.4	15.7	15.6	15.5	15.6
15.4	15.3	16.0	16.0	15.6	16.0	16.1	16.1
15.6	15.6	16.3	16.1	16.0	16.3	16.0	16.0
15.4	15.3	15.2	15.0	15.3	15.3	15.3	15.4
15.4	15.5	15.5	16.0	16.0	16.5	17.0	17.3
16.7	16.7	16.7	17.1	17.1	17.0	17.0	17.3
17.2	17.1	17.0	17.0				

Appendix D.18 Listing of data points for Series 18

EASTERN AIRLINES

8.0	7.7	8.1	8.0	8.0	7.7	7.7	7.7
7.6	7.5	7.5	7.5	8.0	7.7	7.7	8.1
8.2	8.2	8.1	8.1	8.0	8.0	8.0	7.7
7.6	7.6	7.5	7.6	7.6	7.4	7.5	7.5
7.4	7.2	7.1	7.3	7.5	8.0	7.6	7.4
7.4	7.2	7.2	7.2	7.0	7.0	7.0	6.6
7.0	7.3	7.4	7.4	8.4	8.0	8.2	8.1
8.0	7.7	8.4	8.0	8.0	8.0	8.4	8.4
8.4	8.4	8.6	9.0	8.6	8.6	9.0	8.6
8.2	8.4	8.6	8.4	8.6	8.6	8.6	9.0
8.7	8.4	8.2	8.3	8.4	8.6	9.0	9.1
8.6	8.6	9.0	9.1	9.0	9.0	9.1	9.4
9.3	9.3	9.5	9.6				

Appendix D.19 Listing of data points for Series 19*

CHEMICAL CONCENTRATION READING-EVERY TWO HOURS

17.0	16.6	16.3	16.1	17.1	16.9	16.8	17.4
17.1	17.0	16.7	17.4	17.2	17.4	17.4	17.0
17.3	17.2	17.4	16.8	17.1	17.4	17.4	17.5
17.4	17.6	17.4	17.3	17.0	17.8	17.5	18.1
17.5	17.4	17.4	17.1	17.6	17.7	17.4	17.8
17.6	17.5	16.5	17.8	17.3	17.3	17.1	17.4
16.9	17.3	17.6	16.9	16.7	16.8	16.8	17.2
16.8	17.6	17.2	16.6	17.1	16.9	16.6	18.0
17.2	17.3	17.0	16.9	17.3	16.8	17.3	17.4
17.7	16.8	16.9	17.0	16.9	17.0	16.6	16.7
16.8	16.7	16.4	16.5	16.4	16.6	16.5	16.7
16.4	16.4	16.2	16.4	16.3	16.4	17.0	16.9
17.1	17.1	16.7	16.9	16.5	17.2	16.4	17.0
17.0	16.7	16.2	16.6	16.9	16.5	16.6	16.6
17.0	17.1	17.1	16.7	16.8	16.3	16.6	16.8
16.9	17.1	16.8	17.0	17.2	17.3	17.2	17.3
17.2	17.2	17.5	16.9	16.9	16.9	17.0	16.5
16.7	16.8	16.7	16.7	16.6	16.5	17.0	16.7
16.7	16.9	17.4	17.1	17.0	16.8	17.2	17.2
17.4	17.2	16.9	16.8	17.0	17.4	17.2	17.2
17.1	17.1	17.1	17.4	17.2	16.9	16.9	17.0
16.7	16.9	17.3	17.8	17.8	17.6	17.5	17.0
16.9	17.1	17.2	17.4	17.5	17.9	17.0	17.0
17.0	17.2	17.3	17.4	17.4	17.0	18.0	18.2
17.6	17.8	17.7	17.2	17.4			

* Reference [9] pp.525

Appendix D.20 Listing of data points for Series 20*

MONTHLY CHAMPAGNE SALES (IN 1000'S OF BOTTLES) 1962-1970

2.9	2.7	2.8	2.7	2.9	3.0	2.3	2.2
2.9	4.3	5.8	7.1	2.5	2.5	3.0	3.3
3.8	3.2	3.0	1.8	3.6	4.5	6.8	8.4
3.1	3.0	4.0	3.5	3.9	4.0	3.3	1.6
3.5	5.2	7.6	9.3	5.4	3.1	3.7	4.5
4.5	4.5	3.7	1.6	4.7	5.4	8.3	10.7
3.6	4.3	4.2	4.1	4.6	4.8	4.0	1.7
5.0	6.9	9.9	11.3	4.0	4.0	4.5	4.3
5.0	4.7	3.5	1.8	5.2	6.9	10.8	13.9
2.6	2.9	3.4	3.7	2.9	4.0	4.2	1.7
5.2	6.4	9.8	13.1	3.9	3.2	4.3	4.7
5.0	4.9	4.6	1.7	6.0	7.0	9.9	12.7
4.3	3.6	4.6	4.8	4.6	5.3	4.3	1.4
5.9							

* Reference [1]- pp. 273

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